York University
CSE 3101 Fall 2014 – Unit Test 3 of 5
Instructor: Jeff Edmonds

Family Name: ____________________ Given Name: ____________________

Student #: ____________________ Email: ____________________

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1) Quantifiers	20	
2) Sums	20	
3) Dijkstra's Alg	20	
4) Time	20	
5) Network Flow	20	
0) Art	2	
Total	102	

This exam is designed to be completed in 1/2 an hour. Students are expected to complete the exam in that period. If unable to so so, they may continue based on the discretion of the instructor. Keep your answers short and clear.

0 Art therapy question: When half done, draw on the front a picture of how you are feeling.

1. A computational problem is a function P from inputs I to required output $P(I)$. An algorithm is a function A from inputs I to actual output $A(I)$. $Time(A, I)$ gives the running time for algorithm A on input I. Use universal and existential quantifiers (in standard form) to express P is computable time in $3n^2$. How does the prover-adversary game proceed to prove this statement.

2. Consider the following functions.

   - **Classification:** Provide a Θ-approximation, if possible classifying the function into $\Theta(1)$, $2^{\Theta(n)}$, $\log^{\Theta(1)}(n)$, or $n^{\Theta(1)}$. What is this class called?
   - **Summation:** Approximate the sum $\Theta(\sum_{i=1}^{n} f(i))$. Which result did you use and how did you use it? What type of sum is it?

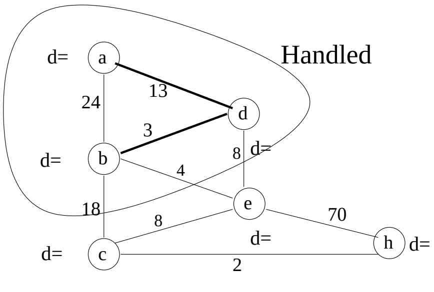
   (a) $f(n) = 5 \cdot n^{5.5} \log^{100}(n)$

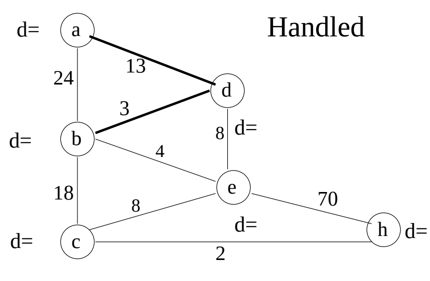
   (b) $f(n) = 5 \cdot 3^{4n} n^{100}$

   (c) $f(n) = 5 + sin(n^2)$

3. Dijkstra's Algorithm:

   (a) Consider a computation of Dijkstra's algorithm on the following graph when the circled nodes have been handled. The start node is a. On the left, give the current values of d. (Be sure to think about what the loop invariant says about the values d.)





   (b) On the right, change the figure to take one step in Dijkstra's algorithm. Include as well any πs that change.

4. Running Time

algorithm $Careful(n)$

$\langle$***pre−cond***$\rangle$**:** n is an integer.

$\langle$***post−cond***$\rangle$**:** $Q(n)$ "Hi"s are printed
for some odd function Q

```
begin
    if( n ≤ 1 )
        PrintHi(1)
    else
        loop i = 1...n
            PrintHi(i)
        end loop
        loop i = 1...8
            Careful(n/2)
        end loop
    end if
end algorithm
```

algorithm $PrintHi(n)$

$\langle$***pre−cond***$\rangle$**:** n is an integer.

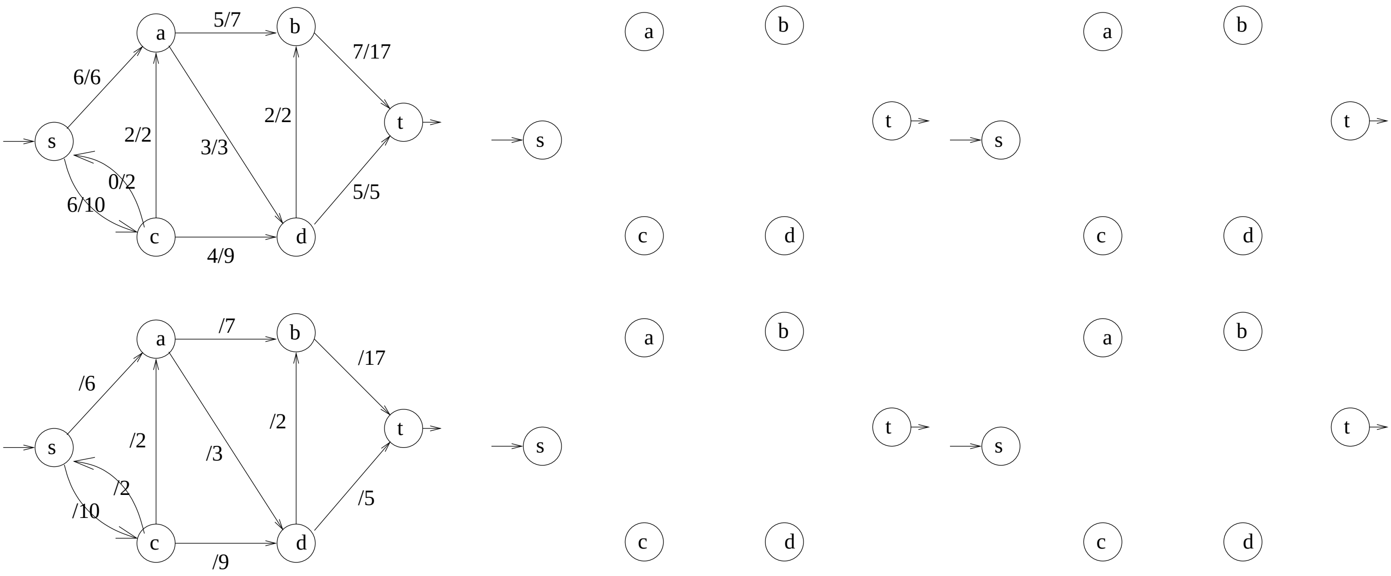
$\langle$***post−cond***$\rangle$**:** n^2 "Hi"s are printed

```
begin
    loop i = 1...n^2
        Print("Hi")
    end loop
end algorithm
```

(a) Give and solve the recurrence relation for the number of "Hi"s $Q(n)$. Show your work. Give a sentence or two giving the intuition.

(b) What is the running time (time complexity) of this algorithm as a function of the size of the input. CAREFUL!!!

5. Network Flow: Consider the following flow network with source s and terminal (or sink) t. The labels on edges are of the form "f/c" where f is the current flow and c is the capacity of the edge. This flow is feasible.



(a) Trace one iteration of the network flow algorithm. Use the nodes in the middle fig to give the augmentation graph for this flow. Make the necessary changes to the left figure to give the new flow.

(b) Prove that there is no flow that is better than this current flow. (Use the right figure if you like.)

(c) Give an algorithm that solves the Min Cut problem and briefly prove that it works. (A total of approximately four sentences.)

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6. (On the exam) NP (Non-Deterministic Polynomial Time)

   (a) What does it mean for a problem P to be in NP?

   (b) What does it mean for a problem P to be NP-Complete.