

High-Performance Multicast Routing in Multi-Channel Multi-Radio Wireless Mesh Networks

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Abstract—Traditional multicast routing algorithms such as shortest path tree (SPT) and minimum Steiner tree (MST) do not consider the wireless broadcast advantage or the underlying channel assignments in a multi-channel multi-radio (MCMR) wireless mesh network (WMN). We propose a multicast routing algorithm for MCMR WMNs that takes into account the above factors in order to minimize the amount of network bandwidth consumed by a routing tree. Experimental results show that routing trees constructed by the proposed algorithm outperform traditional trees such as SPTs, MSTs and minimum number of forwarders trees (MFTs) with respect to packet delivery ratio, throughput and end-to-end delay.

I. INTRODUCTION

Until recently, research on wireless multi-hop networks considers mostly networks with a single channel. The theoretical aggregate throughput for multicasting [1] is estimated as $O(1/\sqrt{n^\epsilon \log n})$, where $0 \leq \epsilon \leq 1$, indicating that the throughput capacity of a single-channel WMN becomes unacceptably low as the network size increases. One of the most effective approaches to enhance the aggregate network throughput is to use systems with multiple radios per node and multiple channels [2].

Traditional multicast routing algorithms such as SPT or Steiner tree do not consider the wireless broadcast advantage (WBA) or the channel assignments (CA) (i.e. channel diversity) in a MCMR WMN. The WBA refers to the fact that the delivery of a data packet from a given node to any number of its neighbors can be done with a single transmission. We propose a multicast routing algorithm that takes into account both the WBA and the channel diversity in order to minimize the amount of network bandwidth consumed by a routing tree. Given a MCMR network and a CA scheme, the algorithm constructs a multicast routing tree that minimizes the total number of transmissions required to deliver a data packet from the source to all multicast destinations. Experimental results show that our routing trees outperform commonly used/cited trees such as SPTs, MSTs and MFTs [3] in terms of packet delivery ratio, throughput and end-to-end delay.

The remainder of the paper is organized as follows. We discuss related work in Section II, and define the problem to be solved in Section III. The proposed algorithm is described in Section IV. In Section V, we present experimental results comparing the performance of the proposed routing trees with that of SPTs, MSTs and MFTs. Section VI concludes the paper and outlines our future work.

II. RELATED WORK

Research work on multicast in *single-channel* WMNs focuses mainly on multicast routing and performance study of routing approaches [3], [4], [5], [6]. The topic of channel assignment and routing for multicast in *multi-channel multi-radio* networks has been studied only recently [7], [8], [9], [10], [11], [12], [13]. The algorithms proposed in [7], [8], [9], [10] aim at minimizing the interference among multicast nodes and maximizing throughput using the “routing first, CA second” approach wherein a multicast routing tree is first constructed, and a CA scheme minimizing interference is then applied to the tree. Using this approach, a node may have more assigned channels than the number of available radios, which requires channel switching. However, currently no channel switching algorithm for multicast is available. Furthermore, channel switching adds considerable delay to data routing in MCMR networks [14].

In this paper, we consider the “CA first, routing second” approach. Given a MCMR network with a pre-determined CA scheme and a multicast group, we construct a multicast routing tree with a minimized number of transmissions, and thus minimize the bandwidth consumption of the tree. Ruiz et al. [3] propose algorithms that build multicast trees with minimized numbers of forwarding nodes, and hence minimized numbers of transmissions, in *single-channel* networks. Our proposed algorithm, on the other hand, minimizes the number of transmissions incurred by a multicast tree in a *multi-channel multi-radio* network. Also using the “CA first, routing second” approach, Lim et al. [11] consider the existing CA to minimize the number of channel conflicts within two-hop distance, but their algorithm requires channel switching.

There exist also algorithms that consider both routing and CA simultaneously [12], [13]. Cheng et al.’s algorithm [12] constructs a multicast tree and a CA scheme with minimized channel conflict and minimal tree cost (defined as the total number of radios used by the nodes in the tree). Chiu et al. [13] propose a CA and tree construction scheme that satisfies a bandwidth constraint. The main limitation of the scheme is the assumption of a *perfect*, no-collision MAC scheduler.

III. PROBLEM DEFINITION

We consider MCMR WMNs with stationary wireless routers (nodes). Two nodes are directly connected and form a communication link if they are within the transmission radio

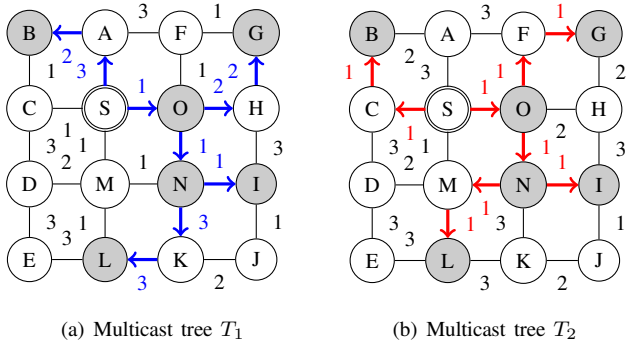


Fig. 1. A network with three channels and two radios per node. Each link is labeled with the assigned channel.

range of each other and share a common channel. We assume that a CA scheme [14], [15], [16], [17], [18], [19], [20] is independently applied to the network prior to the construction of the multicast tree. We also make the following assumptions, which are common constraints imposed by MCMR wireless mesh networking.

- The channels are orthogonal (non-overlapping).
- The channel assigned to a link is used for transmissions in both directions of the link.
- For any node, the number of distinct channels assigned to the node is less than or equal to the number of radios the node possesses. As a result, each radio is bound to a specific channel and no channel switching is needed.

Following is an informal definition of the problem by means of an example. A formal definition can be found in [21].

Consider the MCMR network shown in Fig. 1. Assume that the network has three orthogonal channels and each node has two radios. The number associated with each link indicates the channel assigned to that link by a CA algorithm. (In this example, we use the CA algorithm by Das et al. [18], although the discussion is valid for any CA algorithms with the above assumptions.) Given a multicast group with source S and six destinations B, G, I, L, N and O (shaded nodes), we show two possible routing trees for this multicast group in Fig. 1. The tree nodes are connected by the thick arrows whose directions indicate the data flow in the routing tree. The originating node of each arrow is called a *forwarding node*. The source is considered a forwarding node. A destination can be a forwarding node, e.g., nodes N and O in the above trees.

The problem focuses on the number of transmissions a forwarding node requires to multicast a packet to its one-hop neighbors in the routing tree. For example, in both trees, node N is used as a forwarding node to deliver multicast data to destinations I and L . In tree T_1 (Fig. 1(a)), N has to transmit two copies of every packet (i.e., two transmissions), one on channel 1 to I and the other on channel 3 to K , which will forward the packet to L . However, in tree T_2 (Fig. 1(b)), N needs to perform only one transmission on channel 1 to reach both I and M , which will forward the packet to L . This shows that the choice of route affects the number of transmissions a node has to perform to forward a data packet.

If we sum up the numbers of transmissions that all forwarding nodes in a routing tree T need to perform to deliver a packet to their multicast neighbors in T , the result is the total number of transmissions the tree incurs to deliver a packet from the source to all the destinations, denoted by $S(T)$. (We do not consider retransmissions caused by packet loss or errors.) For the example trees T_1 and T_2 in Fig. 1, $S(T_1) = 9$ while $S(T_2) = 6$. Tree T_2 is preferred because it requires less transmissions per packet and thus consumes less network bandwidth. Among the possible trees connecting the source to the destinations, our goal is to find a tree with the minimum $S(T)$. Finding such a tree is an NP-hard problem [21]. We thus propose a heuristic algorithm to find approximate solutions, which constructs Multi-Channel Minimal Number of transmissions Trees (MCMNTs).

IV. THE MCMNT ALGORITHM

Given a MCMR network with pre-assigned channels, the proposed algorithm operates by increasing the initial solution tree using least cost paths based on link costs. The heuristic works in a similar manner to the Dijkstra's [22] and Prim's algorithms [23] to some extent.

A. Definitions of Link Cost and Path Cost

Consider a connected graph $G = (V, E)$, where V is the set of stationary mesh routers (nodes), and E is the set of communication links (edges) with pre-assigned channels.

- For each node $u \in V$, $\mu_u(c)$ denotes the number of links that are incident on u and assigned channel c . For example, for node A in the graph shown in Fig. 1, $\mu_A(1) = 0$, $\mu_A(2) = 1$ and $\mu_A(3) = 2$. Value $\mu_u(c)$ can be considered as the *channel utilization* of channel c by node u : the higher the value, the more neighbors u can reach with a single transmission on channel c .
- $\mathcal{M}_u = \max_{\forall c \in C} \{\mu_u(c)\}$ is the maximum channel utilization value taken over all channels at node u , where C is the set of available channels.
- $\mathcal{M}_{max} = \max_{\forall u \in V} \{\mathcal{M}_u\}$ which is the maximum \mathcal{M}_u value taken over all nodes u in the network.

Since a *high* channel utilization value is desirable while the heuristic selects paths based on *minimum* costs, we convert channel utilization to a metric whose smaller values are more favorable than higher values in order to perform least cost path selection. In our heuristic, we convert channel utilization value $\mu_u(c)$ of channel c at node u to a new channel metric denoted by $\delta_u(c)$ as follows: $\delta_u(c) = \left(1 + \alpha(\mathcal{M}_{max} - \mu_u(c))\right)$, where $\alpha \geq 0$ is an adjustable parameter.

Each directional link (u, v) is associated with a link cost $w(u, v)$ defined as $w(u, v) = \delta_u^\beta(c) / \delta_v(c)$, where c is the channel used by link (u, v) and $\beta \geq 1$ is an adjustable parameter. The originating node u of the directional link (u, v) is termed the *transmitter*, while the ending node v , the *receiver*. We favor a transmitter with a channel *highly utilized* so that the channel can be used for as many receivers

as possible in the final tree. This explains the term $\delta_u^\beta(c)$ in the link cost.

If a link (u, v) using channel c has been added to the tree, the next link (v, z) to be added should avoid using channel c so that transmissions from u and v do not interfere with each other, because u and v are one-hop neighbors of each other. Therefore, given a transmitter u and an assigned channel c , we should choose a receiver v whose channel c is *lowly utilized* so that node v will have less chance of being selected next as a transmitter *on* channel c . Hence the term $1/\delta_v(c)$ in the link cost. Note also that setting different values for the two parameters α and β may result in different MCMNT multicast trees. Based on empirical testing, we choose $\alpha = 2$ and $\beta = 2$ in the performance evaluation section and will explore other values in future work.

Finally, let $P(s, d)$ denote a path connecting a source s to a destination d . The path cost $W(P(s, d))$ of path $P(s, d)$ is the sum of the costs of the (directional) links on the path. Let $\Phi(s, d)$ be the set of all possible paths connecting s to d . The least cost path $P_m(s, d)$ is defined as the path whose cost is the lowest among all paths in set $\Phi(s, d)$.

Algorithm 1 The MCMNT Algorithm

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1: Input:  $G = (V, E)$ ; source  $s \in V$ ; destination set  $\Delta = \{d_1, \dots, d_m\} \subset V$ ;
2: Output: tree  $T$  connecting  $s$  to  $\Delta$  with minimized  $S(T)$ ; set of forwarding nodes  $F$ .
3: Other global variables: current set of unconnected destinations  $\Delta_{cur}$ ; current set of forwarding nodes  $F_{cur}$ ; current tree  $T_{cur}$ .
4: Initialization:  $\Delta_{cur} = \Delta$ ;  $F_{cur} = \{s\}$ ;  $T_{cur} = \{s, \emptyset\}$ ; compute the costs  $w$  of all directional links in  $E$ ;
5: START
6: while  $\Delta_{cur} \neq \emptyset$  do
7:    $P_{min} = \text{NULL}$ ; {least cost path (LCP) in this round}
8:    $W(P_{min}) = \infty$ ; {cost of this path}
9:    $d_{min} = \text{NULL}$ ; {destination of this LCP}
   {Find an unconnected destination that can be connected to the current tree with the minimum cost.}
10:  for all nodes  $v \in T_{cur}$  do
11:    Compute the LCP connecting  $v$  to each node in  $\Delta_{cur}$  using Dijkstra's algorithm.
12:    Among these LCPs, select the path  $P(v, d)$  with the smallest cost, where  $d$  is some node in  $\Delta_{cur}$ .
    {Keep  $P(v, d)$  if it is better than current  $P_{min}$ }
13:    if  $W(P(v, d)) < W(P_{min})$  then
14:       $P_{min} = P(v, d)$ ;  $d_{min} = d$ ;
15:    end if
16:  end for
  { $d_{min}$  can be connected to the current tree with the minimum cost among the unconnected destinations. Add  $d_{min}$  and  $P_{min}$  to tree.}
17:   $T_{cur} = T_{cur} \cup \{\text{nodes and links on } P_{min}\}$ ;
18:   $F_{cur} = F_{cur} \cup \{\text{intermediate nodes on } P_{min}\}$ ;
19:   $\Delta_{cur} = \Delta_{cur} \setminus d_{min}$ ;
  {Update applicable link costs to take advantage of the WBA in the next round.}
20:  for all link  $(u, v)$  in  $P_{min}$  do
21:     $\{N_u$  denotes the set of one-hop neighbors of a node  $u\}$ 
22:    for all  $z \in N_u$  do
23:      if  $z \notin T_{cur}$  and  $\text{channel}(u, z) = \text{channel}(u, v)$  then
24:         $w(u, z) = 0$  {link cost set to zero}
25:      end if
26:    end for
27:  end for
28: end while {terminates when all destinations are connected to  $T_{cur}$ }
29:  $T = T_{cur}$ ;  $F = F_{cur}$ ; return  $[T, F]$ ;
30: END

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B. The Algorithm

The proposed MCMNT heuristic is summarized by the above **Algorithm 1**. Initially, the initial solution tree consists only the source, s . Multicast destinations are then added to the tree one by one using the least cost path from each destination to the current tree (the **while** loop on line 6). In particular, for each node v in the current tree T_{cur} , we find the least cost path connecting v to each node d in the current set of unconnected destinations Δ_{cur} using the Dijkstra's algorithm (line 11). We then consider all the computed least cost paths $P(v, d)$, $\forall d \in \Delta_{cur}$, $\forall v \in T_{cur}$, and select the path P_{min} with the minimum cost (lines 13-14). This path P_{min} and the corresponding destination d_{min} are then added to the solution tree (lines 17-19).

We then update the applicable link costs to take advantage of the WBA in the next round of inserting a new destination to the tree (lines 20-27). Specifically, for each directional link (u, v) on path P_{min} just selected, and for each one-hop neighbor z of u that currently resides *outside* the tree, if link (u, z) is assigned the same channel as link (u, v) , we update the cost of link (u, z) to zero (lines 23-24). By doing this, we increase the chance of link (u, z) being selected in the next round. If (u, z) is later added to the solution tree, u will be able to reach z without requiring one more transmission. These link cost updates aim at exploiting the WBA, as suggested by Wieselthier et al. [24]. The above procedure is repeated until all the destinations are added to the solution tree. The time complexity¹ of **Algorithm 1** is $O(|\Delta||V|^2)$ as proved in [21].

V. EXPERIMENTAL RESULTS

Using the QualNet simulator version 4.0 [25], we compare the performance of MCMNTs with that of SPTs, MSTs [26] and MFTs. We simulate a medium-size MCMR network of 100 wireless routers uniformly distributed over a 1700m \times 1700m area with random channel assignments. Each wireless router (node) has a transmission range of 350m. We use PHY802.11b at the physical layer with a data rate of 11 Mbits/s. The IEEE802.11 CSMA/CA protocol without RTS/CTS exchange is chosen as the multicast medium access control protocol. At the transport layer, we do not use any flow or congestion control mechanisms in order to test the network performance under very high loads. The packet size excluding the headers is 512 bytes. Each multicast group has one source placed at the center sending data at a constant bit rate (CBR), while the destinations are randomly scattered around the network. (Destinations are wireless routers of the WMN backbone.) In each experiment, the source transmits at a specified CBR for 600 seconds of simulated time. The simulator then continues to run for 100 seconds of simulated time to give the last packets time to be routed. Each data point in the graphs is averaged from five runs using different random seeds and plotted with a confidence interval of 95%.

¹The asymptotic bound remains the same as in [21], but we have since fine tuned and improved the performance of the algorithm described in [21].

For each type of tree, we measure the total number of transmissions per packet $S(T)$ (as defined in Section III), average packet delivery ratio (PDR), average throughput, and average packet end-to-end delay (averaged over all destinations) as functions of

- *multicast group size*. The number of multicast destinations varies from 20 to 80 nodes. The number of radios per node and the number of available channels are set to 3. The source transmits at a rate of 200 packets/s.
- *multicast traffic load*. The multicast source rate at the application layer varies from 100 to 300 packets/s. The number of channels is 3 and the group size is 40.
- *number of channels*. The number of channels is set to 1, 2, 3, 5, and 7. The group consists of 40 destinations and the multicast rate is set to 200 packets/s.

A. Function of Group Size

The results of this set of experiments are shown in Fig. 2. The graph in Fig. 2(a) confirms that the MCMNT tree requires the least number of transmissions, followed by the MFT, MST and SPT in that order. For example, the number of transmissions incurred by the MCMNT in the 80-destination tree is 22%, 42% and 42% less than by the MFT, MST and SPT, respectively. For all types of trees, as the group size increases, more forwarding nodes are added and the number of transmissions per packet goes up, as expected.

The graph in Fig. 2(b) shows that the MCMNTs offer the highest PDRs in all cases. For instance, the MCMNT PDR is 10%, 15%, and 32% higher than those of the SPT, MST and MFT, respectively, when there are 20 destinations. The performance gap between the MCMNT and the MST/MFT narrows down as the group size increases. However, the MCMNT PDR is still significantly higher than the PDRs of the other trees. In Fig. 2(c), a similar trend is observed for the average throughput. For example, for a group of size 20, the MCMNT offers 15%, 25% and 57% higher throughput than the SPT, MST and MFT, respectively. When a forwarding node n uses less channels (i.e., less transmissions) to multicast a data packet, that reduces the probability of packet collision with the packets its neighboring nodes transmitting on n 's unused channels. That explains the higher PDRs of the MCMNTs.

The MCMNT also incurs the lowest end-to-end delay in most cases, as shown in Fig. 2(d). When a forwarding node uses more channels to multicast a data packet, more neighboring nodes may have to defer their transmissions if they also use the same channels, resulting in higher packet end-to-end delay. The MCMNT's low end-to-end delay is a result of less channels being used by a forwarding node.

B. Function of Traffic Load

The graph in Fig. 3(a) confirms that the MCMNT requires the least number of transmissions, 19%, 35% and 39% less than the MFT, MST and SPT, respectively. The PDR and throughput of the MCMNT are higher than those of the other trees in most cases, especially under high traffic loads (Fig. 3(b) and 3(c)). A lower number of transmissions enables

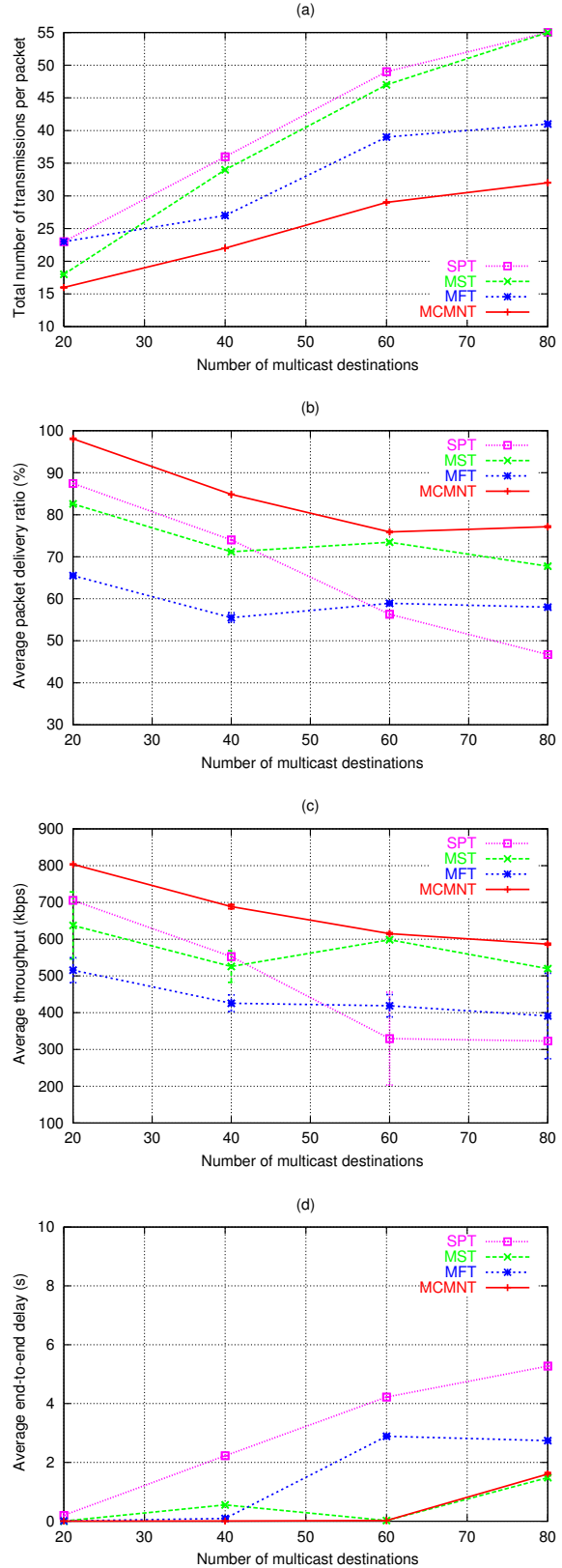


Fig. 2. Functions of group size (3 channels; source rate 200 packets/s)

the MCMNT to achieve better performance, as discussed earlier. The MCMNT also has the lowest average end-to-end delay thanks to the least number of transmissions resulting in less contention among nodes in the routing tree (Fig. 3(d)).

C. Function of Number of Channels

In this set of experiments, we vary the number of orthogonal channels from one to seven. In general, increasing the number of channels improves the average PDRs, throughputs and end-to-end delays of all trees, as illustrated by the graphs in Fig. 4. Note that as the number of channels increases, the number of transmissions also goes up in many cases (Fig. 4(a)). More transmissions in this case, however, do not necessarily imply performance degradation, because the loads are distributed over more channels and parallel transmissions can be used with less interference. That explains the improved performance as the number of channels increases from one to seven.

When only one channel is available, the MFT has the least number of transmissions since the MFT algorithm is optimized for single-channel networks. As a result, the MFT provides the best PDR, throughput and end-to-end delay in this special case (Fig. 4(b), 4(c) and 4(d)). However, when multiple channels are used, the MCMNT algorithm produces trees with the least numbers of transmissions and, consequently, the highest PDRs and throughputs, as well as the lowest end-to-end delays. We also observe that the performance gap between the MCMNTs and the other trees narrows as the number of channels goes up. The reason is that data packets are distributed over a larger number of channels, making the problem of minimizing interference (or the number of transmissions) less relevant. Nevertheless, MCMNTs still offer noticeably better performance than the other trees, especially with respect to PDR and throughput.

To confirm the results of the above three sets of experiments, we created several configurations for each data point by varying the node placement in the network and selecting different multicast sources and destinations. We also ran experiments with networks of other sizes (50 and 200 nodes). The results from these experiments are consistent with those presented in this paper.

VI. CONCLUSION

We study the problem of building multicast routing trees with minimum numbers of transmissions in WMNs where multiple channels and multiple radios are used. The objective is to minimize interference among multicast nodes for improved performance. Our experimental results show that MCMNTs perform significantly better than commonly used/cited trees such as SPTs, MSTs and MFTs in terms of PDR, throughput and end-to-end delay. Our current and future work on this problem includes the following: (1) fine-tuning the adjustable parameters α and β for optimal performance; (2) designing and evaluating distributed implementations of the MCMNT heuristic; and (3) incorporating the current traffic load at each node into the link and path costs for better load balancing and performance under dynamic network conditions.

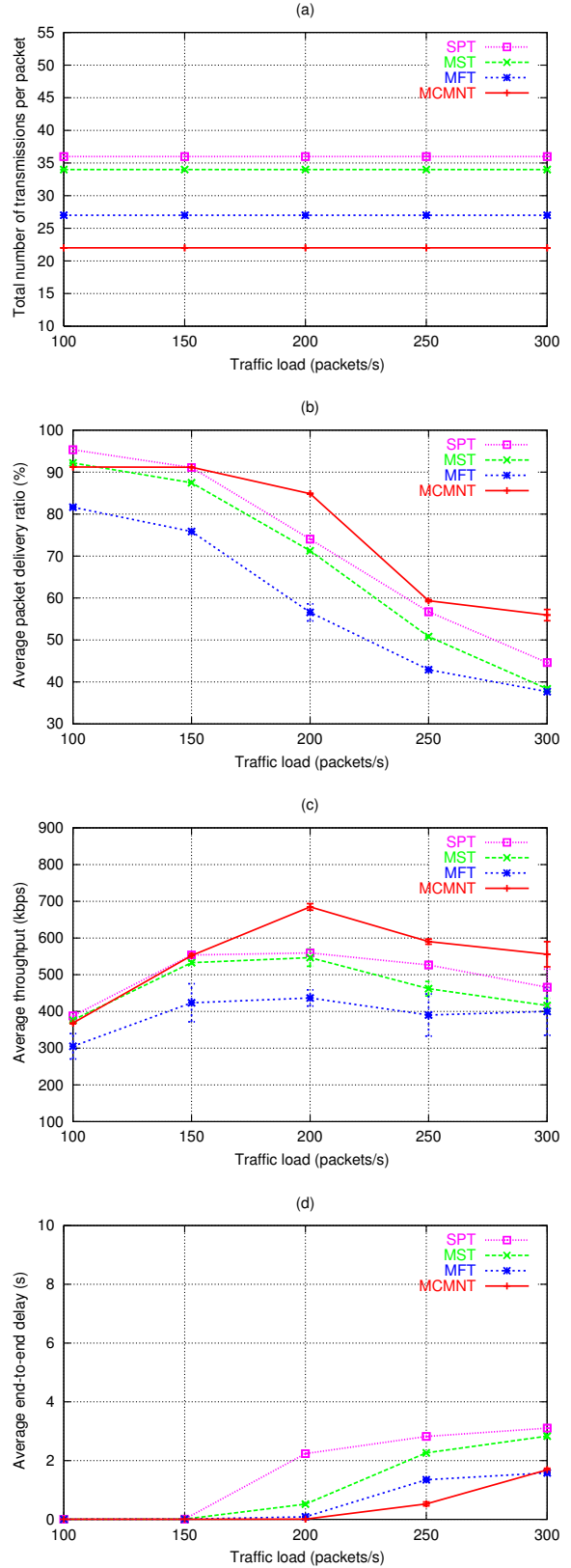


Fig. 3. Functions of traffic load (3 channels; 40 destinations)

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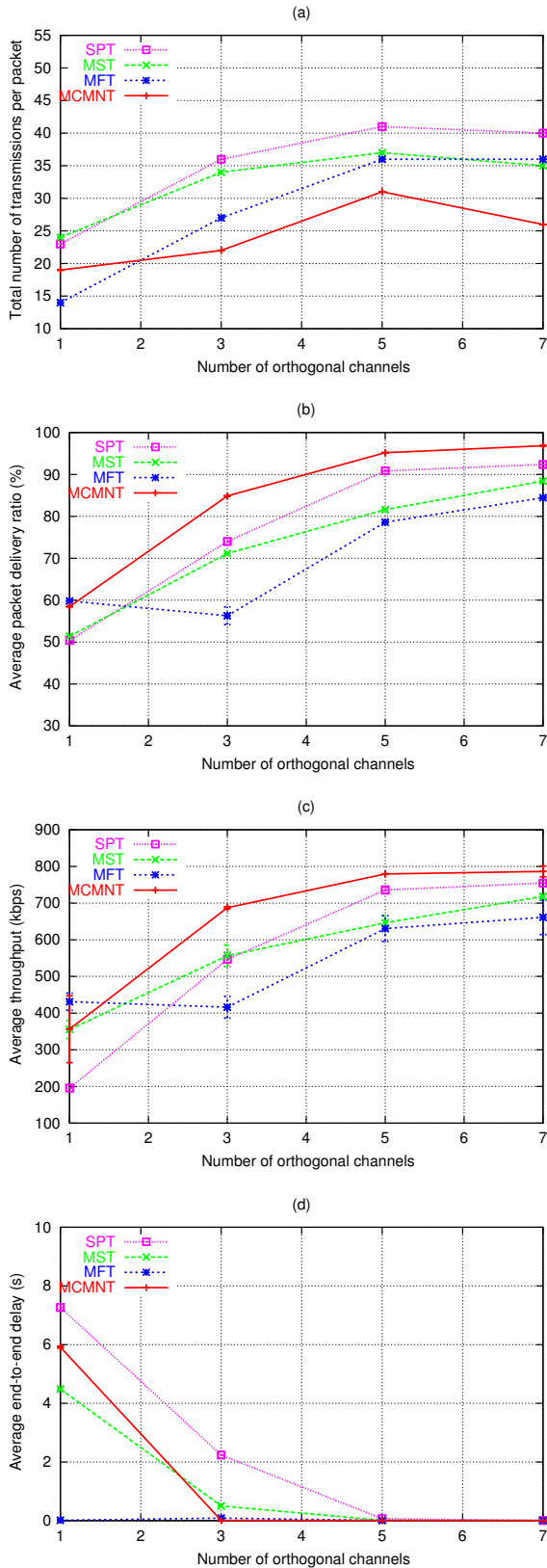


Fig. 4. Functions of number of channels (source rate 200 packets/s; 40 destinations; one radio per node for one channel; 2 radios per node for 2 channels; 3 radios per node for 3, 5 and 7 channels)

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