Stereo EKF Pose-based SLAM for AUVs

9-th Open German-Russian Workshop on PATTERN RECOGNITION and IMAGE UNDERSTANDING

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Problem Statement

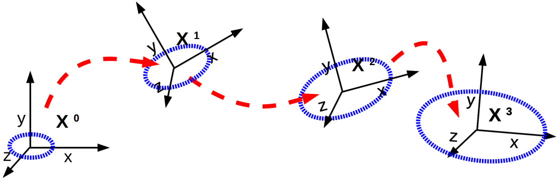
Problem Statement

- Accessibility of the sub-aquatic world is important for research and industry
- AUV[1] promising advantages compared to ROV[2]
    - Untethered, independent, self-powered, ...
- Question: **How to perform the localization** of AUVs
- Localization task becomes a crucial issue in AUVs
    - significant errors can lead to the mission failure

[1] Autonomous Underwater Vehicle
[2] Remotely Operated Vehicle

SLAM

- **Vehicle State** = pose (Position and Orientation)
- **State Vector** = collection of Vehicle States
- Visual Odometry
  - Displacement of two consecutive Images
  - Estimation of the Relative Motion
  - Prone to drift
- Periodical adjustment is necessary
- SLAM (Simultaneous Localization And Mapping)
  - Identification of already visited environment needed
  - Refines pose of landmarks of environment
- Extended Kalman Filtering (EKF)



Related Work

Related Work

- [Schattschneider et al., 2011]
  - Underwater SLAM
  - Stereo Camera System used for ship hull inspection
  - 3D Landmarks used to detect Loop Closings
  - State = [poses , landmarks]

- [Eustice et al., 2008]
  - Underwater SLAM
  - Landmarks not saved in X
  - But: Image Registration used at every Iteration
  - State = [linear velocity, acceleration and angular rate]

Related Work

- [This study]
  - Stereo Camera System (pure 3D data)
  - Orientations represented in the quaternion space
  - Image Registration used at every $n-$Iteration
  - State = [poses]

3D Transformation

3D Transformation

- Classical Transformation for 6 DOF
  - composition $\oplus$
  - inversion $\ominus$

- Jacobian Matrices $J_{\oplus}$ and $J_{\ominus}$
  - Robot Transformation is non-linear
  - Direct Covariance computation in not possible
  - Approximation: Linearisation of transformation functions

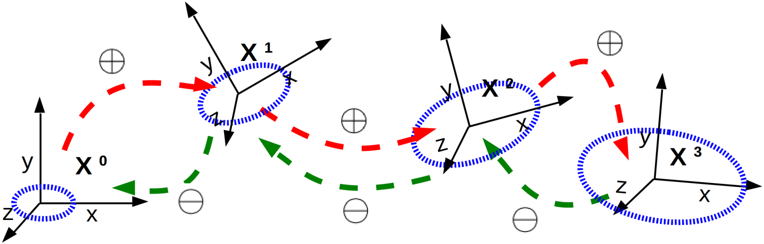
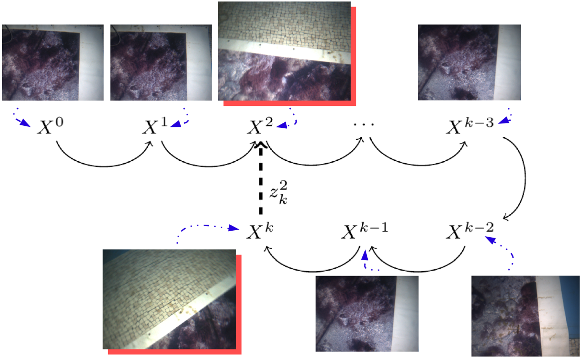


Image Registration

Image Registration

- Verifies if two stereo images close a loop
  - Different time instants, view points, height, environmental conditions
  - ! certain overlap

result 3D camera Transformation between two images $z_k = [R, t]$



Pseudocode

Algorithm 1: Image Registration

input : Current Stereo Image pair S_l, S_r and Recorded Image I_c candidate to close a loop with

output: 3D Transformation $[R, t]$

begin

  $F_c \leftarrow$ `findFeature` (I_c);

  $[F_l, F_r] \leftarrow$ `stereoMatching` (S_l, S_r);

  if `match` $(F_l, F_c) == true$ **then**

    $[F_l, F_r] \leftarrow$ `updateFeature` (F_l, F_r);

    $P_{3D} \leftarrow$ `calc3DPoints` (F_l, F_r);

    $[R, t] \leftarrow$ `solvePnPRansac` (F_c, P_{3D});

    return $[R, t]$

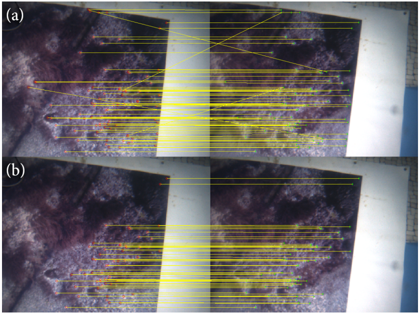
  else

    return error;

stereoMatching(S_l, S_r)

- First: $[F_l, F_r] = findFeature(S_l, S_r)$
- Second: Comparing the squared differences of F_l and F_r
  - Differences reaches a certain treshold $\hookrightarrow$ Matched
- Usage of RANSAC

result 2 sets of matching Feature Descriptors $[F_l, F_r]$



Pseudocode

Algorithm 2: Image Registration

input : Current Stereo Image pair S_l, S_r and Recorded Image I_c candidate to close a loop with

output: 3D Transformation $[R, t]$

begin

$[F_l, F_r] \leftarrow$ `stereoMatching` (S_l, S_r);

$F_c \leftarrow$ `findFeature` (I_c);

if `match` $(F_l, F_c) == true$ **then**

  $[F_l, F_r] \leftarrow$ `updateFeature` (F_l, F_r);

  $P_{3D} \leftarrow$ `calc3DPoints` (F_l, F_r);

  $[R, t] \leftarrow$ `solvePnPRansac` (F_c, P_{3D});

  return $[R, t]$

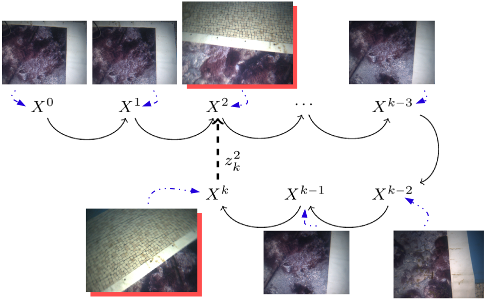
else

  return error;

solvePnPRansac(F_c, P_{3D})

- Solves the Perspective N-Point Problem (PnP)
- Estimates a pose transformation
- Minimizes the Reprojection Error between
  - 3D Feature
  - corresponding 2D Feature

result 3D Transformation $z_k = [R, t]$



EKF-SLAM

EKF-SLAM

- State-Estimation of non-linear system
  - Using normally distributed Gaussian noise
- Three Stages
  1. Prediction Stage
  2. State Augmentation Stage
  3. Update Stage

Algorithm 2: Visual EKF-SLAM

input : $X, C, O, C_o, S_l, S_r, C_m\ I_n$
output: Updated state vector X_u, covariance C_u and recorded Images I_u

begin
  ; `/* Prediction stage */`
  $X_t \leftarrow$ `getLastState` (X) ;
  $C_t \leftarrow$ `getLastCovariance` (C) ;
  $[X_t^+, C_t^+] \leftarrow$ `composition` (X_t, C_t, O, C_o) ;
  ; `/* Augmentation stage */`
  $X^+ \leftarrow$ `addState` (X, X_t^+);
  $C^+ \leftarrow$ `addCovariance` (C, C_t^+);
  ; `/* Update stage */`
  $z \leftarrow$ `imageRegistration` (S_l, S_r, I_n);
  if `imageRegistration` == *false* **then**
    return;
  else
    $[h, H] \leftarrow$ `calcHkK` (X^+, z) ;
    $y \leftarrow$ `innovation` (h, z) ;
    $S \leftarrow$ `innovationCov` (C^+, H, C_m) ;
    $K \leftarrow C^+ \cdot H^T \cdot S^{-1}$;
    $X_u \leftarrow X^+ + K \cdot y_k$;
    $C_u \leftarrow (1 - K \cdot H) \cdot C^+$;
  $I_u \leftarrow I_n \bigcup S_l$;
end

composition(X_t, C_t, O, C_o)

- Performs Composition $\oplus$
  - $X_+ = X_t \oplus O$
- Calculates Covariance Matrix
  - $C_+ = J_{1\oplus} \cdot C_t \cdot J_{1\oplus}^T + J_{2\oplus} \cdot C_o \cdot J_{2\oplus}^T$

EKF-SLAM

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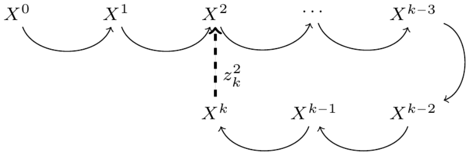
$I_u \leftarrow I_n \bigcup S_l$;
end

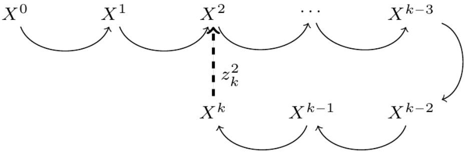
Update Stage

- Dependent on *Image Registration*
- No *Image Registration* $\hookrightarrow$ No Update
- Corrects State Vector



$calcHkK(X^+, z)$

- *observation function h*
  - Based on z relative motions from X are calculated
  - $h_k = \ominus X^k \oplus X^2$
  - Comparable h_k (State Vector) and z_k (Image Registration)
  - Multiple Loop Closings $h = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_k \end{bmatrix}$



innovation(h, z)

- In general: Difference between h and z
  - $y = z - h$
- **Translation**: subtraction
- Due to quaternions special treatment necessary
  - Different quaternions - similar orientation
  - Solution: Absolute values $\hookrightarrow y_q = |q_z| - |q_h|$
- **Rotation**: subtracting the modules

Pseudocode

Algorithm 2: Visual EKF-SLAM

input : $X, C, O, C_o, S_l, S_r, C_m\ I_n$
output: Updated state vector X_u, covariance C_u and recorded Images I_u

begin
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- ; `/* Update stage */`
- $z \leftarrow$ `imageRegistration` (S_l, S_r, I_n);
- **if** `imageRegistration` $==$ *false* **then**
  - return;
- **else**
  - $[h, H] \leftarrow$ `calcHkK` (X^+, z) ;
  - $y \leftarrow$ `innovation` (h, z) ;
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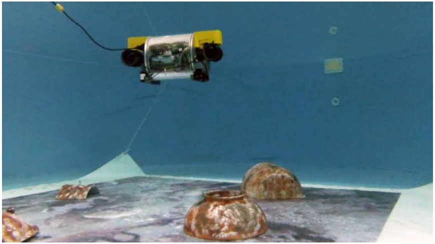
  - $K \leftarrow C^+ \cdot H^T \cdot S^{-1}$;
  - $X_u \leftarrow X^+ + K \cdot y_k$;
  - $C_u \leftarrow (1 - K \cdot H) \cdot C^+$;
- $I_u \leftarrow I_n \bigcup S_l$;

end

Experimental Results

Experimental Results

- System
  - Laptop (Intel core i7 (2×2.9Ghz), 8GB RAM and SSD)
  - Ubuntu 12.04, MATLAB R2013a (single CPU core used)
- Set-Up
  - Fugu-C (Bumblebee 2 1032×776 pixel)
  - Watertank inside the UIB ($7\text{m} \times 4\text{m} \times 1.5\text{m}$)
- Ground Truth: printed digital image of a Seabed



Experimental Results

- Test
  - 23.42m sweeping task
  - 6 noise levels
- Error Definition:
  - Difference between
    - Ground Truth $\leftrightarrow$ Odometry
    - Ground Truth $\leftrightarrow$ EKF-SLAM
  - Divided by the length of the Trajectory (Ground Truth)
  - Error units are meters per travelled meter

Experimental Results

- Quantitative Results

Noise Level *Covariance*	*1* *0*	*2* *3e-9*	*3* *9e-9*	*4* *3e-8*	*5* *5e-7*	*6* *3e-6*
Odom. error ∅	0.038	0.417	0.494	0.806	2.614	6.898
EKF error ∅	0.027	0.282	0.285	0.309	0.590	0.953
Improv. (%)	28.9	32.3	42.3	61.6	77.4	86.1

Figure: Comparison between visual odometry and EKF-SLAM trajectory mean error (∅) with respect to the ground truth. Error is measured in meters per traveled meter.

Experimental Results

- Quantitative Results
  - Image Registration used at every $n-$Iteration
  - Separation of 4 already faster than Mission-Time

Separation between frames	*2*	*4*	*8*
Run-Time (min)	8.4	4.3	2.3
error (m)	0.28	0.32	0.39

Figure: Comparison run time of different key-frame separations and error. Used noise level 2.

Experimental Results

- Qualitative Results Blue: Ground Truth, Black: Odometry, Red: EKF-SLAM
  - Noise Level 2

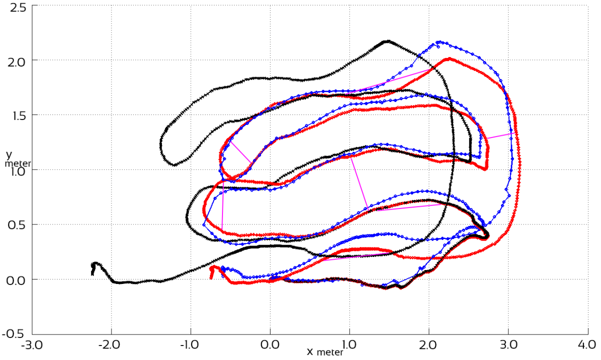


Figure: Example result with a noise level of two. Additionally the eight loop closings are plotted (magenta lines).

Experimental Results

- Qualitative Results Blue: Ground Truth, Black: Odometry, Red: EKF-SLAM
  - Noise Level 4

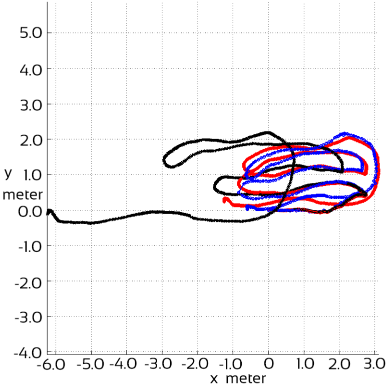


Figure: Example result with a noise level of four.

Experimental Results

- Qualitative Results Blue: Ground Truth, Black: Odometry, Red: EKF-SLAM
  - Noise Level 6

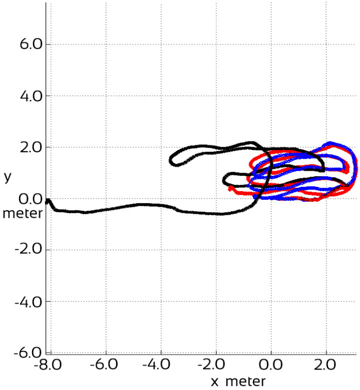


Figure: Example result with a noise level of six.

Conclusion

Conclusion

- Summary
  - Pose based visual EKF-SLAM approach
  - Generic Solution for vehicles with up to 6 DOF (theoretically)
  - Only Stereo Camera Data
  - Orientation is represented in the quaternion space
  - state vector $X = [poses]$
  - Considerably localization correction
  - With Separation of 4 Execution-Time already under Mission-Time

Literature I

Eustice, R. M., Pizarro, O., and Singh, H. (2008).
Visually Augmented Navigation for Autonomous Underwater Vehicles.
Ieee Journal Oceanic Engineering, 33:103–122.

Schattschneider, R., Maurino, G., and Wang, W. (2011).
Towards stereo vision SLAM based pose estimation for ship hull inspection.
Oceans 2011, pages 1–8.

- The whole bibliography is listed in the corresponding paper

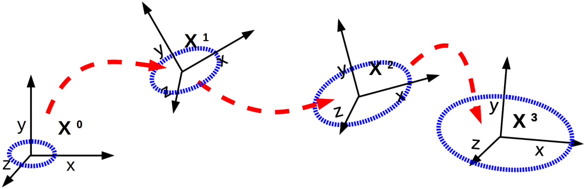
Appendix

Autonomous Underwater Vehicles (AUVs)

- Remotely Operated Vehicles (ROVs)
    - Tethered
    - Support Vessels
    - Limited operative range
- Autonomous Underwater Vehicles
    - (Try to) Overcome this limitations
    - Highly repetitive, long or hazardous missions
    - Self-Powered
    - Independent (support ships and weather)
    - Reduction of
        - missions costs
        - human resources
        - execution time

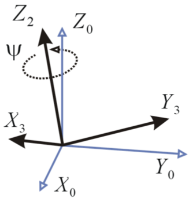
Vehicle Localization

- Several possibilities
- Using:
  - IMU (velocity, orientation, and gravitational forces)
  - Odometry (Acoustic Sensors or Cameras)
  - Sensor Fusion
- Prone to Drift
- Visual Odometry, because Cameras
  + Spatial and Temporal Resolution
  + More Environmental Data
  - Dependent on light and visibility

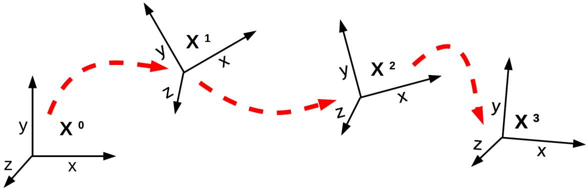


Vehicle Localization

- **pose** = Position and Orientation
- 6 Degrees of Freedom
    - 3 Translation
    - 3 Rotation



- Vehicle State X = pose (in this work)
- collection of poses = **State Vector** $\hookrightarrow$ **Trajectory**



SLAM

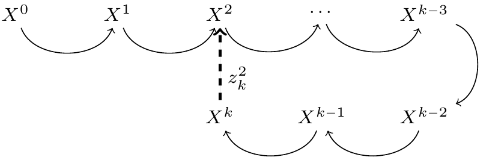
- Visual Odometry
  - Displacement of two consecutive Images
  - **Estimation** of the Absolute Motion (Prone to drift)
- SLAM (Simultaneous Localization And Mapping)
  - Most successful approach
  - Computes pose
  - Refines pose of landmarks of environment
- Extended Kalman Filtering (EKF)

= **Visual EKF SLAM**

Displacement in x-Direction

EKF (In a Nutshell)

- Three Stages
  1. Prediction Stage
     - Predicting vehicle's localization (visual odometry)
     - Prone to drift
     - Uncertainty is modelled with covariance matrix
  2. State Augmentation Stage
     - Prediction is added to the end of X
     - Uncertainty accumulates over time
  3. Update Stage
     - Detection of Loop Closings
     - Provide the system with more reliable Data
     - Update X



Applications

- Maintenance
- Rescue Operations
- Surveying
- Infrastructure Inspections
- Sampling

Related Work

- Literature is scarce, but deals mainly with:
  - Correcting the odometry with the result of the Image Registration
  - Adding Landmarks to X
    - + Continous Correction of pose and landmarks
    - + Whole X is corrected
    - – Increasing complexity over time (X gets big)
    - – On-line usage no longer possible

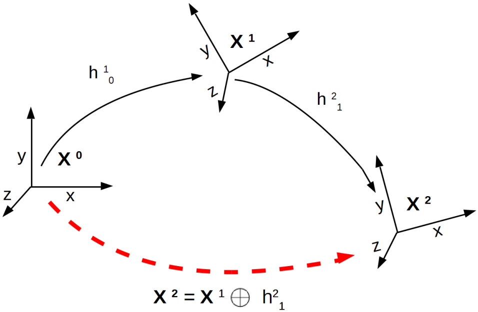
Composition $\oplus$

- Composition: $X_+ = \begin{bmatrix} X_+^t \\ X_+^r \end{bmatrix}$
- Quaternions (Orientation)
  - $q = \begin{bmatrix} q_w & q_1 & q_2 & q_3 \end{bmatrix}$
  - faster computation
  - no trigonometric functions
  - no gimbal lock
- $X_+^r = q^T \cdot q^P$

$$A = \begin{bmatrix} -2 \cdot q_2^2 - 2 \cdot q_3^2 + 1 & 2 \cdot q_1 \cdot q_2 - 2 \cdot q_3 \cdot q_w & 2 \cdot q_1 \cdot q_3 + 2 \cdot q_2 \cdot q_w & 0 \\ 2 \cdot q_1 \cdot q_2 + 2 \cdot q_3 \cdot q_w & -2 \cdot q_1^2 - 2 \cdot q_3^2 + 1 & 2 \cdot q_2 \cdot q_3 - 2 \cdot q_1 \cdot q_w & 0 \\ 2 \cdot q_1 \cdot q_3 - 2 \cdot q_2 \cdot q_w & 2 \cdot q_2 \cdot q_3 + 2 \cdot q_1 \cdot q_w & -2 \cdot q_1^2 - 2 \cdot q_2^2 + 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Composition $\oplus$

- Composition: $X_{+} = \begin{bmatrix} X_{+}^{t} \\ X_{+}^{r} \end{bmatrix}$
- $X_{+}^{t} = \begin{bmatrix} x^{P} \\ y^{P} \\ z^{P} \\ 1 \end{bmatrix} + A^{P} \cdot \begin{bmatrix} x^{T} \\ y^{T} \\ z^{T} \\ 1 \end{bmatrix}$



Inversion $\ominus$

- Task: Invert $T = \left[\underbrace{x, y, z}_{t} \quad \underbrace{q_w, q_1, q_2, q_3}_{A}\right]$

$$\begin{array}{cccc} \vec{n} & \vec{o} & \vec{a} & \vec{p} \end{array}$$
$$\begin{pmatrix} & A & & t \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} & A & & t \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} & & & -\vec{n} \circ \vec{p} \\ & A^T & & -\vec{o} \circ \vec{p} \\ & & & -\vec{a} \circ \vec{p} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- Result is

$$\ominus X = \begin{bmatrix} -\vec{n} \circ \vec{p} \\ -\vec{o} \circ \vec{p} \\ -\vec{a} \circ \vec{p} \\ q^{-1\,T} \end{bmatrix}$$

Jacobian Matrices $J_{1\oplus}$, $J_{2\oplus}$ and $J_{\ominus}$

- $J_{1\oplus}$ and $J_{2\oplus}$
  - Composition $\oplus$ has two parameters (T and P)
  - Each: Jacobian Matrix of $X_+ \hookrightarrow J_{1\oplus}$ and $J_{2\oplus}$

$$J_{1\oplus} = \begin{bmatrix} 1 & 0 & 0 & 2\cdot q_2^X\cdot z^Y - 2\cdot q_3^X\cdot y^Y & 2\cdot q_2^X\cdot y^Y + 2\cdot q_3^X\cdot z^Y & 2\cdot q_1^X\cdot y^Y - 4\cdot q_2^X\cdot x^Y + 2\cdot q_w^X\cdot z^Y & 2\cdot q_1^X\cdot z^Y - 2\cdot q_w^X\cdot y^Y - 4\cdot q_3^X\cdot x^Y \\ 0 & 1 & 0 & 2\cdot q_3^X\cdot x^Y - 2\cdot q_1^X\cdot z^Y & 2\cdot q_2^X\cdot x^Y - 4\cdot q_1^X\cdot y^Y - 2\cdot q_w^X\cdot z^Y & 2\cdot q_1^X\cdot x^Y + 2\cdot q_3^X\cdot z^Y & 2\cdot q_w^X\cdot x^Y - 4\cdot q_3^X\cdot y^Y + 2\cdot q_2^X\cdot z^Y \\ 0 & 0 & 1 & 2\cdot q_1^X\cdot y^Y - 2\cdot q_2^X\cdot x^Y & 2\cdot q_3^X\cdot x^Y + 2\cdot q_w^X\cdot y^Y - 4\cdot q_1^X\cdot z^Y & 2\cdot q_3^X\cdot y^Y - 2\cdot q_w^X\cdot x^Y - 4\cdot q_2^X\cdot z^Y & 2\cdot q_1^X\cdot x^Y + 2\cdot q_2^X\cdot y^Y \\ 0 & 0 & 0 & q_w^Y & -q_1^Y & -q_2^Y & -q_3^Y \\ 0 & 0 & 0 & q_1^Y & q_w^Y & q_3^Y & -q_2^Y \\ 0 & 0 & 0 & q_2^Y & -q_3^Y & q_w^Y & q_1^Y \\ 0 & 0 & 0 & q_3^Y & q_2^Y & -q_1^Y & q_w^Y \end{bmatrix}$$

- Covariance of Composition $\oplus$:

$$C_+ = J_{1\oplus}\cdot C^1\cdot J_{1\oplus}^T + J_{2\oplus}\cdot C^2\cdot J_{2\oplus}^T$$

Jacobian Matrices $J_{1\oplus}$, $J_{2\oplus}$ and $J_{\ominus}$

- $J_{\ominus}$
  - Composition $\ominus$ has one parameter
  - Derivation will give us $J_{\ominus}$

$$J_{\ominus} = \begin{bmatrix} 2\cdot q_2^X\cdot q_2^X + 2\cdot q_3^X\cdot q_3^X - 1 & -2\cdot q_1^X\cdot q_2^X - 2\cdot q_3^X\cdot q_w^X & 2\cdot q_2^X\cdot q_w^X - 2\cdot q_1^X\cdot q_3^X & 2\cdot z^X\cdot q_2^X - 2\cdot y^X\cdot q_3^X & -2\cdot y^X\cdot q_2^X - 2\cdot z^X\cdot q_3^X & 4\cdot x^X\cdot q_2^X - 2\cdot y^X\cdot q_1^X + 2\cdot z^X\cdot q_w^X & 4\cdot x^X\cdot q_3^X - 2\cdot y^X\cdot q_w^X - 2\cdot z^X\cdot q_1^X \\ 2\cdot q_3^X\cdot q_w^X - 2\cdot q_1^X\cdot q_2^X & 2\cdot q_1^X\cdot q_1^X + 2\cdot q_3^X\cdot q_3^X - 1 & -2\cdot q_2^X\cdot q_3^X - 2\cdot q_1^X\cdot q_w^X & 2\cdot x^X\cdot q_3^X - 2\cdot z^X\cdot q_1^X & 4\cdot y^X\cdot q_1^X - 2\cdot x^X\cdot q_2^X - 2\cdot z^X\cdot q_w^X & -2\cdot x^X\cdot q_1^X - 2\cdot z^X\cdot q_3^X & 2\cdot x^X\cdot q_w^X + 4\cdot y^X\cdot q_3^X - 2\cdot z^X\cdot q_2^X \\ -2\cdot q_1^X\cdot q_3^X - 2\cdot q_2^X\cdot q_w^X & 2\cdot q_1^X\cdot q_w^X - 2\cdot q_2^X\cdot q_3^X & 2\cdot q_1^X\cdot q_1^X + 2\cdot q_2^X\cdot q_2^X - 1 & 2\cdot y^X\cdot q_1^X - 2\cdot x^X\cdot q_2^X & 2\cdot y^X\cdot q_w^X - 2\cdot x^X\cdot q_3^X + 4\cdot z^X\cdot q_1^X & 4\cdot z^X\cdot q_2^X - 2\cdot y^X\cdot q_3^X - 2\cdot x^X\cdot q_w^X & -2\cdot x^X\cdot q_1^X - 2\cdot y^X\cdot q_2^X \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

- Covariance of Inversion $\ominus$:

$$C_{-} = J_{\ominus} \cdot C \cdot J_{\ominus}^{T}$$

calc3DPoints(F_l, F_r)

- Result: 3D Points
- Missing depth-value z can be calculated
    - Reprojection Matrix Q

$$Q = \begin{bmatrix} 1 & 0 & 0 & -C_x \\ 0 & 1 & 0 & -C_y \\ 0 & 0 & 0 & f_x \\ 0 & 0 & -\frac{1}{T_x} & \frac{(C_x - C_{x'})}{T_x} \end{bmatrix}$$

- C_x and C_y optical center
- f_x focal length
- $T_x = \text{baseline} \cdot f_x$
- Primed from left Camera, unprimed from right Camera

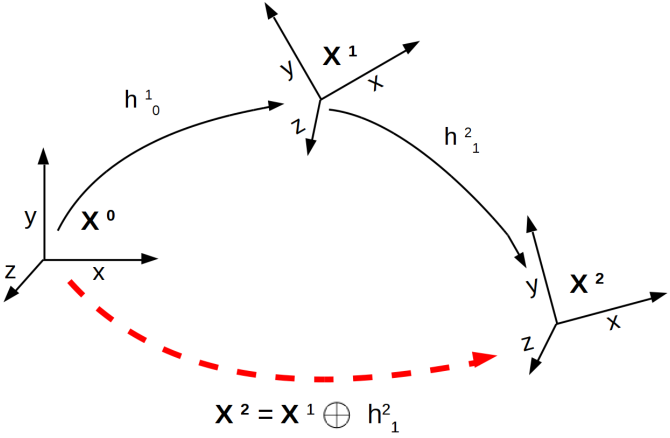
result 3D Points P_{3D}

Composition $\oplus$

Composition $\oplus$

- Adds a relative Transformation h to an absolute State X^x

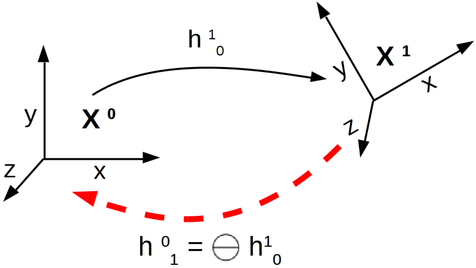
result new absolute pose X_+



Inversion $\ominus$

Inversion $\ominus$

- Inverts a Transformation h
- With $\oplus$ used to get relative Transformations from absolutes

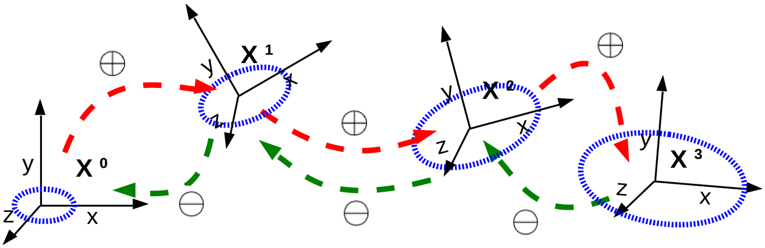


Jacobian Matrices $J_{1\oplus}$, $J_{2\oplus}$ and $J_{\ominus}$

- Necessary to compute the uncertainty
- Apply: Taylor Series of first order

= **Covariance**: Uncertainty with zero mean random Gaussian noise

- Jacobian for each Transformation $\oplus$ and $\ominus$



Exemplary result

- With respect to the Pseudocode
  - S_l is transformed into I_c (if overlap big enough)
  - Transformation is done in 3D

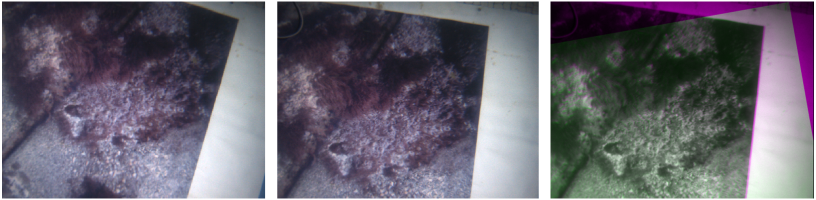


Figure: Left: S_l; middle: loop closing image I_c. On the right: the transformation of the image registration applied to S_l. The purple color indicates the error of the transformation.

getLastState(X)

- Takes the last 7 Elements of X

$$X = \Big[\underbrace{x^1 \quad y^1 \quad z^1 \quad q_w^1 \quad q_1^1 \quad q_2^1 \quad q_3^1}_{\text{vehicle pose at } 1^{\text{st}} \text{ iteration}} \quad \cdots \quad \underbrace{x^n \quad y^n \quad z^n \quad q_w^n \quad q_1^n \quad q_2^n \quad q_3^n}_{\text{vehicle pose at } n^{\text{th}} \text{ iteration}}\Big]^T$$

addCovariance$(C,\ C_t^+)$

- Not only adding (true for diagonal)

$$C = \begin{bmatrix} \begin{bmatrix} \sigma_{11}^1 & & & & & & \\ & \sigma_{22}^1 & & & & \alpha & \\ & & \sigma_{33}^1 & & & & \\ & & & \sigma_{44}^1 & & & \\ & & & & \sigma_{55}^1 & & \\ & \beta & & & & \sigma_{66}^1 & \\ & & & & & & \sigma_{77}^1 \end{bmatrix} & \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \ddots & \mathbf{D} \\ \mathbf{E} & \mathbf{F} & \begin{bmatrix} \sigma_{11}^n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{22}^n & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{33}^n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{44}^n & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{55}^n & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{66}^n & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{77}^n \end{bmatrix} \end{bmatrix}$$

- Except for diagonal
- e.q. **B** and **E** are calculated
  - $\mathbf{B} = \mathbf{A} \cdot J_{1\oplus}^T$
  - $\mathbf{E} = J_{1\oplus} \cdot \mathbf{C}$

getLastCovariance(C)

- Takes the last 7×7 Matrix of C

$$C = \begin{bmatrix} \begin{bmatrix} \sigma_{11}^1 & & & & & & \\ & \sigma_{22}^1 & & & \alpha & & \\ & & \sigma_{33}^1 & & & & \\ & & & \sigma_{44}^1 & & & \\ & & & & \sigma_{55}^1 & & \\ & \beta & & & & \sigma_{66}^1 & \\ & & & & & & \sigma_{77}^1 \end{bmatrix} & \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \ddots & \mathbf{D} \\ \mathbf{E} & \mathbf{F} & \begin{bmatrix} \sigma_{11}^n & & & & & & \\ & \sigma_{22}^n & & & & \delta & \\ & & \sigma_{33}^n & & & & \\ & & & \sigma_{44}^n & & & \\ & & & & \sigma_{55}^n & & \\ & \gamma & & & & \sigma_{66}^n & \\ & & & & & & \sigma_{77}^n \end{bmatrix} \end{bmatrix}$$

$calcHkK(S_l, S_r, I_n)$

- *observation matrix H*
  - Stores Jacobian Matrices
  - Partially derivatives of h with respect to X^+
  - Elements of H not referring to used states are 0

innovation(h, z)

- $q_z = [0.996, -0.010, 0.014, 0.083]$ $(1.55^\circ,\ -1.38^\circ,\ 9.50^\circ)$
- $q_h = [-0.996, -0.018, 0.001, -0.083]$ $(0.04^\circ,\ 2.09^\circ,\ 9.55^\circ)$
- $y_q = q_z - q_h = [1.992, 0.007, 0.013, 0.166]$
- Solution: Absolute values
- $y_q = |q_z| - |q_h|$
- $y_q = [0.0000, -0.0073, 0.0134, -0.0003]$

innovation(h, z)

- $y_q = q_z - q_h = [1.99274, 0.007344, 0.013427, 0.166257]$
- Pure subtraction: Big innovation (not right!)
- Solution: Absolute values
- $y_q = |q_z| - |q_h|$
- $y_q = [0.0000, -0.0073, 0.0134, -0.0003]$

$innovationCov(C^+, H\ C_m)$

- $S = H \cdot C \cdot H^T + R$
- *Measurement Matrix* R
- Size of R depends on number of detected Loop Closings

$$R = \begin{bmatrix} C_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & C_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & C_m \end{bmatrix} \tag{1}$$

Experimental Results

- Quantitative Results
  - Noise level increases $\leftrightarrow$ Improvement by EKF-SLAM increases

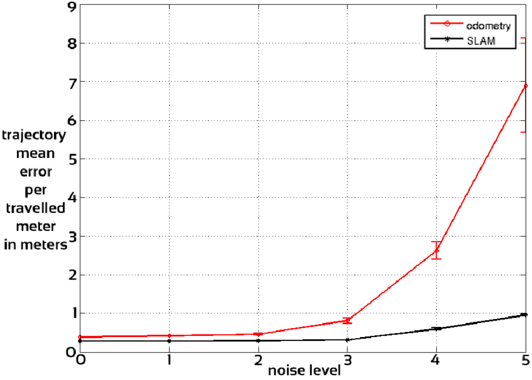


Figure: Comparison between state mean errors using raw odometry and EKF pose estimates. The standard deviation is set to 0.1σ.