

# Composition of ConGolog Programs

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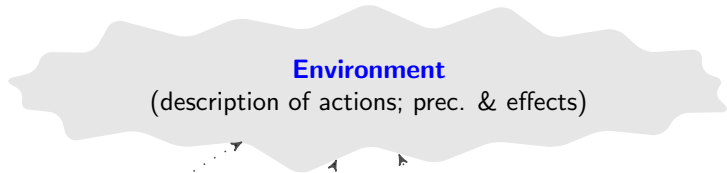
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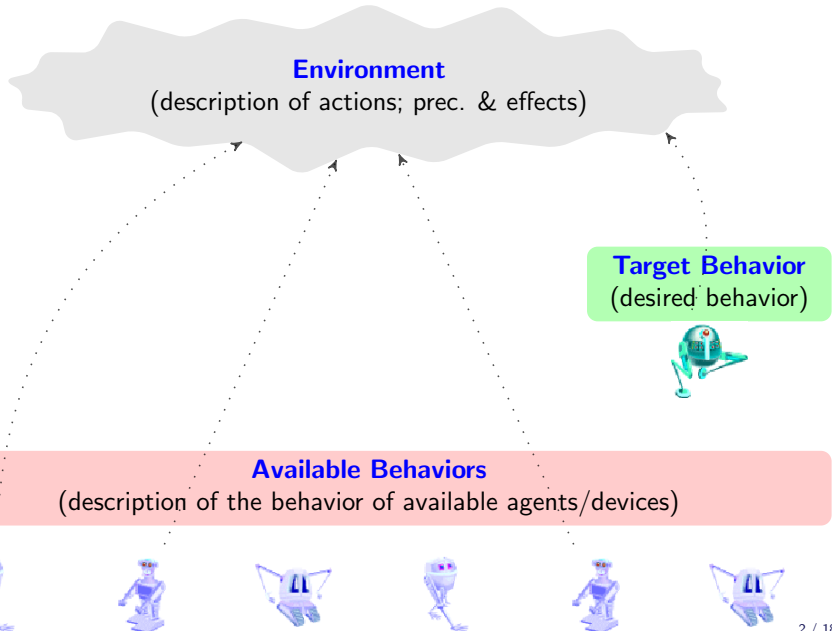
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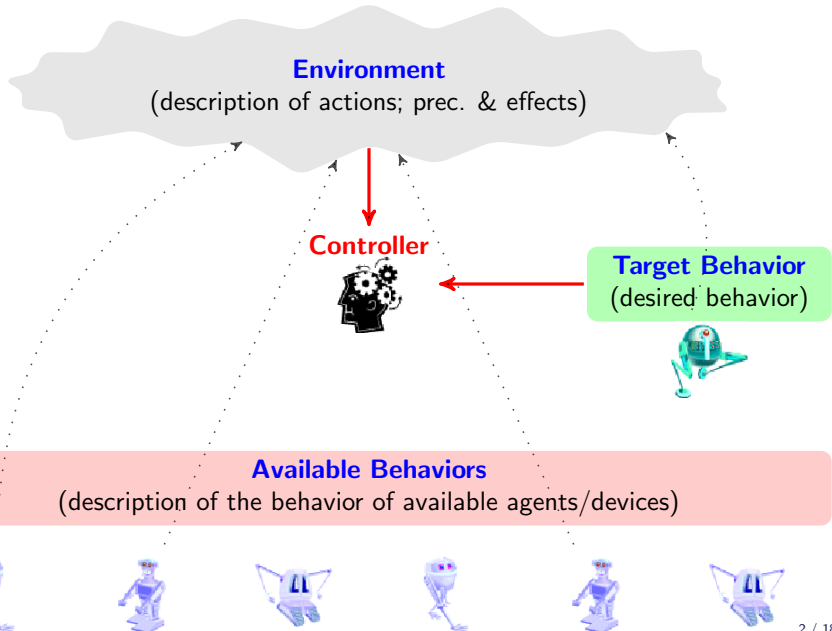
**Available Behaviors**  
(description of the behavior of available agents/devices)



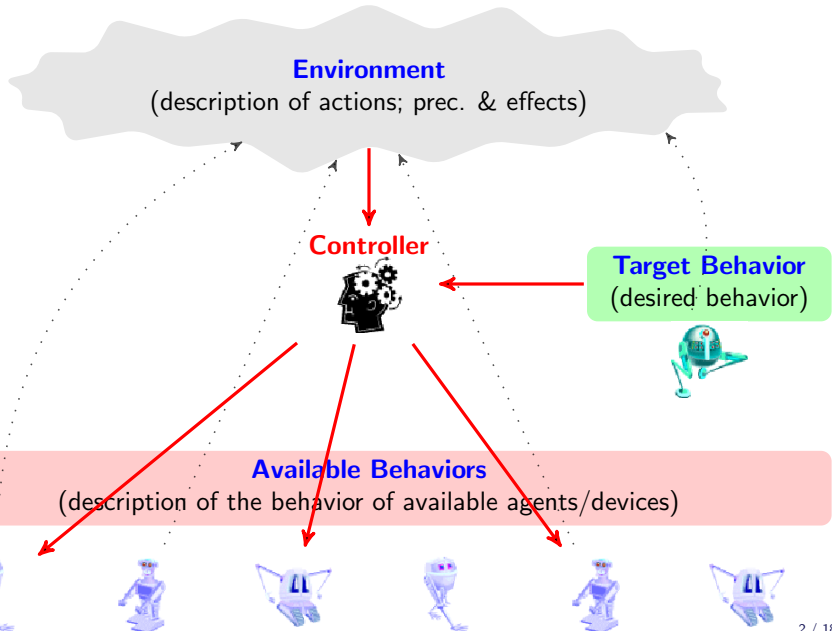
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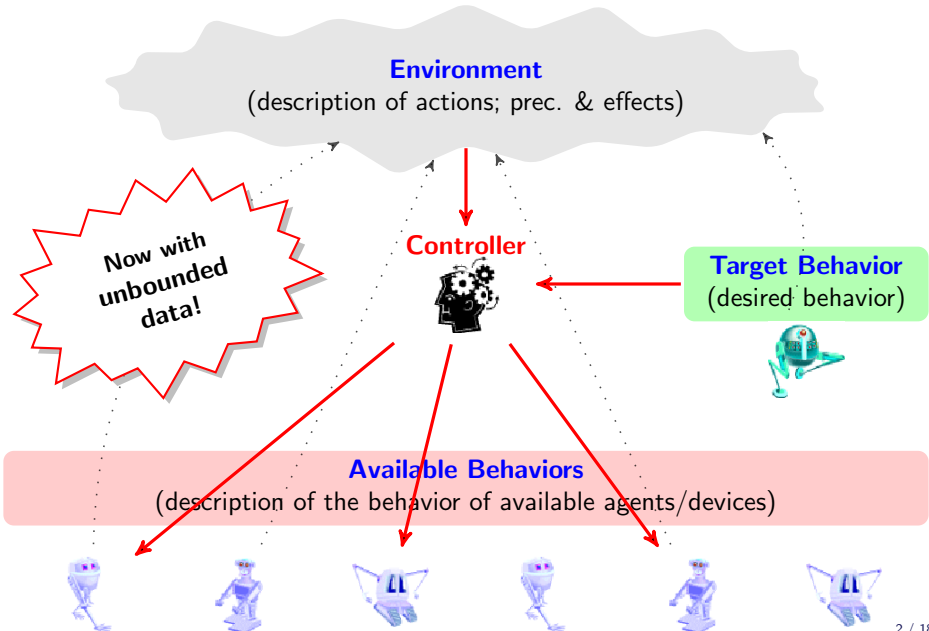
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## Notable features:

- Programs may include *non-deterministic* points & may *not terminate*.
- Domain may be *infinite*.
- Programs may go over *infinite states*.

# Controller for a Music Jukebox

```
while True do {  
  if ( $\neg$ Playing  $\wedge$  ( $\exists$ song)Pending(song)) then  
    ( $\pi$  song, disk).{  
      (Pending(song)  $\wedge$  InDisk(song, disk))?;  
      select(song);  
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- Domain tests relative to an action theory.
- Domain actions.
- Nondeterministic features.

# Semantics for High-Level Programs

In terms of two predicates:

- 1  $Trans(\delta, s, \delta', s')$ : program  $\delta$  can *evolve one step* from situation  $s$  to situation  $s'$  with remaining program  $\delta'$ .

$$Trans(\delta_1; \delta_2, s, \delta', s') \equiv Trans(\delta_1, s, \delta'_1, s') \wedge \delta' = \delta'_1; \delta_2 \vee Final(\delta_1, s) \wedge Trans(\delta_2, s, \delta', s').$$

- 2  $Final(\delta, s)$ : program  $\delta$  *may terminate successfully* in  $s$ .

$$Final(\delta_1; \delta_2, s) \equiv Final(\delta_1, s) \wedge Final(\delta_2, s)$$

# Formalizing the Composition Problem: Simulation

Informally:

System  $S$  *simulates* system  $T$  if  $S$  can “match” all  $T$ ’s moves, forever.



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## Formally [Milner IJCAI'71]:

Given two labelled TSs  $S = (\Sigma_S, A_S, \longrightarrow_S)$  and  $T = (\Sigma_T, A_T, \longrightarrow_T)$ , the simulation is the largest relation  $\text{Sim} \subseteq \Sigma_S \times \Sigma_T$  such that:

if  $\text{Sim}(s, t)$  holds (state  $s$  simulates state  $t$ ), then:

if  $t \xrightarrow{\alpha}_T t'$ , then

there exists  $s \xrightarrow{\alpha}_S s'$  and  $\text{Sim}(s', t')$ .

# The Composition Problem: Simulation

$Sim(\delta_t, \delta_1, \dots, \delta_n, s)$ : available programs can *simulate* the target program in  $s$ .

$$Sim(\delta_t, \delta_1, \dots, \delta_n, s) \equiv \\ \exists S. (S(\delta_t, \delta_1, \dots, \delta_n, s) \wedge \forall \delta_t, \delta_1, \dots, \delta_n, s. \Theta[S](\delta_t, \delta_1, \dots, \delta_n, s)),$$

where

$$\Theta[S](\delta_t, \delta_1, \dots, \delta_n, s) \stackrel{\text{def}}{=} \\ S(\delta_t, \delta_1, \dots, \delta_n, s) \rightarrow \\ (Final(\delta_t, s) \rightarrow \bigwedge_{i=1, \dots, n} Final(\delta_i, s)) \wedge \\ (\forall \delta'_t, s' Trans(\delta_t, s, \delta'_t, s') \rightarrow \\ \bigvee_{i=1, \dots, n} \exists \delta'_i. Trans(\delta_i, s, \delta'_i, s') \wedge S(\delta'_t, \delta_1, \dots, \delta'_i, \dots, \delta_n, s')).$$

If the simulation holds then one can build an **orchestrator generator** based on it.

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# The Technique

Relies on the following “tools” / notions:

**1 Simulation approximates:**

[Tarski '55]

- Check simulation in a finite way.

**2 Regression mechanism:**

[Reiter'91; Pirri & Reiter'99]

- Reason on formulas after action performance.

**3 Characteristic graphs:**

[Classen&Lakemeyer KR'08]

- Abstract (infinite) program states into a finite graph.

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# Simulation Approximates

$Sim_k(\delta_t, \delta_1, \dots, \delta_n, s)$ :

the available programs can “simulate”  $k$  steps of the target program in  $s$ .

$$Sim_0(\delta_t, \delta_1, \dots, \delta_n, s) \equiv (Final(\delta_t, s) \rightarrow \bigwedge_{i=1, \dots, n} Final(\delta_i, s)).$$

$$\begin{aligned} Sim_{k+1}(\delta_t, \delta_1, \dots, \delta_n, s) \equiv & \\ & Sim_k(\delta_t, \delta_1, \dots, \delta_n, s) \wedge \\ & (\forall \delta'_t, s'. Trans(\delta_t, s, \delta'_t, s') \rightarrow \\ & \bigvee_{i=1, \dots, n} \exists \delta'_i. Trans(\delta_i, s, \delta'_i, s') \wedge Sim_k(\delta'_t, \delta_1, \dots, \delta'_i, \dots, \delta_n, s')). \end{aligned}$$

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## Proposition

For every  $k \geq 0$ , if

$$Sim_k(\delta_t, \delta_1, \dots, \delta_n, s) \equiv Sim_{k+1}(\delta_t, \delta_1, \dots, \delta_n, s),$$

then

$$Sim_k(\delta_t, \delta_1, \dots, \delta_n, s) \equiv Sim(\delta_t, \delta_1, \dots, \delta_n, s).$$

# The Technique

Relies on the following “tools” / notions:

## 1 Simulation approximates:

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## 2 Regression mechanism:

[Reiter'91; Pirri & Reiter'99]

- Reason on formulas after action performance.
- computes what has to be true in situation  $s$  so that  $\phi$  is true after doing action  $\alpha$  in  $s$ .
- $\mathcal{R}[\phi(do(\alpha, s))] = \phi'(s)$       action  $\alpha$  has been eliminated!

## 3 Characteristic graphs:

[Classen&Lakemeyer KR'08]

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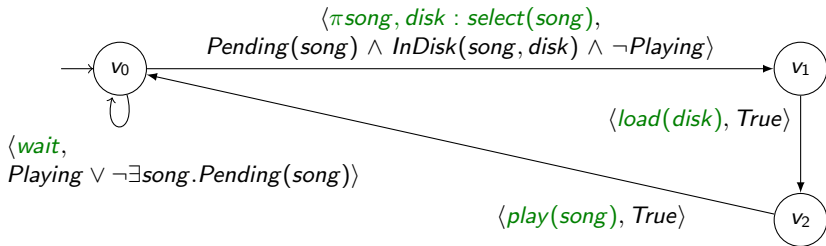
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# Characteristic Graph for $\delta_{music}$



```

 $\delta_{music} \doteq$ 
while True do {
  if ( $\neg Playing \wedge (\exists song) Pending(song)$ ) then
     $\pi song, disk.$ {
      ( $Pending(song) \wedge InDisk(song, disk)$ );
      select(song);
      load(disk);
      play(song)
    }
  else wait
}

```

$v_0 = \langle \delta_{music}, False \rangle$

$v_1 = \langle load(disk); play(song), False \rangle$

$v_2 = \langle play(song), False \rangle$

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If the simulation holds then one can build an **orchestrator generator** based on it.

## Algorithm SYMSIM( $\delta_t^0, \delta_1^0, \dots, \delta_n^0$ )

Computes relation  $X$  containing tuples of the form  $\langle v_t, v_1, \dots, v_n, \phi \rangle$ :

- $v_t, v_1, \dots, v_n$  are nodes in the characteristic graphs.
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$$\mathbf{1} \quad X_0 := \{ \langle (\delta_t, \gamma_t), (\delta_1, \gamma_1), \dots, (\delta_n, \gamma_n), \gamma_t \rightarrow \bigwedge_{i=1}^n \gamma_i \rangle \mid (\delta_j, \gamma_j) \text{ in } \mathcal{G}_{\delta_j^0} \}$$

- 0-step simulation: check for termination "mimicking."

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2 At every step, compute NEXT[X]: “one step refinement” of the simulation:

$$\text{NEXT}[X] = \{ \langle v_t, v_1, \dots, v_n, \phi_{old} \wedge \phi_{new} \rangle \mid \langle v_t, v_1, \dots, v_n, \phi_{old} \rangle \in X \}.$$

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4 Stop when  $X = \text{NEXT}[X]$ .



## NEXT[X]: One-step refinement of the simulation

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For every potential target evolution from  $v_t$  to  $v'_t$  via action  $\alpha_t$ , ...

if it can be done in the program ( $\psi_t$  holds) and the action  $\alpha_t$  is possible, ...

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For every potential target evolution from  $v_t$  to  $v'_t$  via action  $\alpha_t$ , ...

if it can be done in the program ( $\psi_t$  holds) and the action  $\alpha_t$  is possible, ...

then some available prog.  $\delta_i$  can evolve from  $v_i$  to  $v'_i$  via action  $\alpha_i$  such that:

- 1 the action  $\alpha_i$  can be matched to  $\alpha_t$ ;
- 2 the program can indeed do the step;
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# Technical Results

## Theorem

If algorithm  $\text{SYM}\text{SIM}(\delta_t^0, \delta_1^0, \dots, \delta_n^0)$  terminates returning the set  $X$ . Then,

$$\text{Axioms} \models \text{Sim}(\delta_t, \delta_1, \dots, \delta_n, s) \equiv \phi[s],$$

where  $\langle (\delta_t, \gamma_t), (\delta_1, \gamma_1), \dots, (\delta_n, \gamma_n), \phi \rangle \in X$ .



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Moreover, we can construct a **delegator controller** to realize the composition *on-the-fly* using FO entailment only (on  $\mathcal{D}_{S_0}$ ).

Idea:

after a request, jump to a configuration  $\langle v_t, v_1, \dots, v_n, \phi \rangle \in X$  for which  $\phi$  holds.

# Conclusions

Reasoning on unbounded data and processes is a challenge for CS since it leads *infinite* state systems.

- Standard approach: abstract to finite systems (see literature on Verification).
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Based on transforming the **second-order formula for checking the dynamic property of simulation** into a **first-order one talking only about the static properties of the initial situation/DB**.

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## Specifically:

Based on transforming the **second-order formula for checking the dynamic property of simulation** into a **first-order one talking only about the static properties of the initial situation/DB**.

## Main research direction for future work:

- 1 incomplete information about the initial situation.
  - *offline* vs *online* interpreters (see literature on high-level programs in AI).
- 2 identify cases in which the technique becomes sound & complete.