The Gaussian Pyramid

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In this note the construction of the Gaussian pyramid is reviewed; for more details see [2, 3, 4]. The Gaussian pyramid consists of low-pass filtered, reduced density (i.e., downsampled) images of the preceding level of the pyramid, where the base level is defined as the original image. More formally, let the two-dimensional original image be denoted by $I(x, y)$. The Gaussian pyramid is defined recursively as follows,

$$G_0(x, y) = I(x, y), \text{ for level, } l = 0$$
$$G_l(x, y) = \sum_{m=-2}^{2} \sum_{n=-2}^{2} w(m, n) G_{l-1}(2x + m, 2y + n), \text{ otherwise}$$

where $w(m, n)$ is a weighting function (identical at all levels) termed the *generating kernel* which adheres to the following properties: separable, symmetric and each node at level n contributes the same total weight to nodes at level $l+1$. An example of $w(m, n)$ is the 5-tap filter: $\frac{1}{16}[\ 1 \quad 4 \quad 6 \quad 4 \quad 1\ ]$. The weighting function closely approximates the Gaussian function, hence the origins of the pyramids name. Alternatively, the same result can be realized by applying an *equivalent weighting function* denoted $w_l(m, n)$ (unique for each level l) directly to the original image, followed by l downsampling operations, where l denotes the level number. The *equivalent weighting function* approximates a Gaussian function that doubles in scale with each level. In the frequency domain the filter's passband at level l is one octave lower than its predecessor level at $l - 1$. For an image of dimensions N-by-N the total number of operations (consisting of additions and multiplications) to generate the full pyramid is $7N^2$ [3].

An alternative view of the Gaussian pyramid is that each element of the pyramid represents a local average obtained with the *equivalent weighting function* applied to the original image. Thus the Gaussian pyramid contains local averages at various scales [3]. This particular view has been leveraged for texture analysis [5] and target localization [1].

References

[1] C.H. Anderson, P.J. Burt, and G.S. van der Wal. Change detection and tracking using pyramid transform techniques. In *SPIE Conference on Intelligent Robotics and Computer Vision*, pages 72–78, 1985.

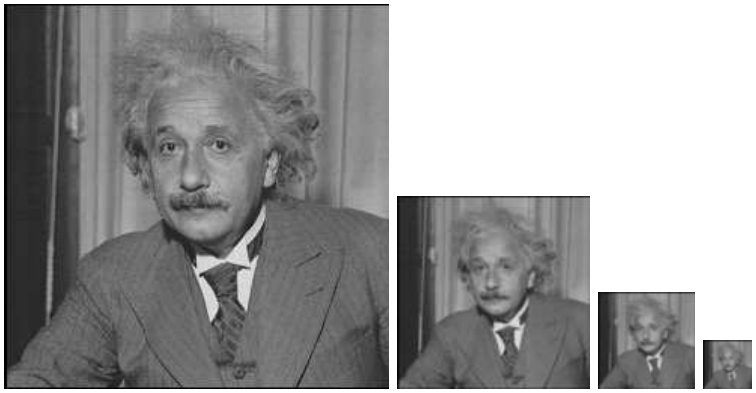

Figure 1: Gaussian Pyramid. Depicted are four levels of the Gaussian pyamid, levels 0 to 3 presented from left to right.

[2] P.J. Burt. Fast filter transforms for image processing. *Computer Graphics and Image Processing*, 1981.

[3] P.J. Burt. Fast algorithms for estimating local image properties. *Computer Graphics and Image Processing*, 1983.

[4] P.J. Burt and E.H. Adelson. The laplacian pyramid as a compact image code. *IEEE Transactions on Communication*, 31(4):532–540, April 1983.

[5] L.I. Larkin and P.J. Burt. Multi-resolution texture energy measures. In *IEEE Conference on Computer Vision and Pattern Recognition*, 1983.