Fourier Definitions

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Fourier series of a periodic continuous-time signal:

$$x(t) = \sum_{n=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t} \qquad (\textit{synthesis equation}) \tag{1}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk(2\pi/T)t} dt \qquad (\textit{analysis equation}) \tag{2}$$

Discrete-time Fourier series for a periodic discrete-time signal:

$$x[n] = \sum_{n=0}^{N-1} a_k e^{jk(2\pi/N)n} \qquad (\textit{synthesis equation}) \tag{3}$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk(2\pi/N)n} \qquad (\textit{analysis equation}) \tag{4}$$

Fourier transform of an aperiodic continuous-time signal:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega \qquad (\textit{synthesis equation}) \tag{5}$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt \qquad (\textit{analysis equation}) \tag{6}$$

Discrete-time Fourier transform (DTFT) of an aperiodic continuous-time signal:

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega \qquad (\textit{synthesis equation}) \tag{7}$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n} \qquad (\textit{analysis equation}) \tag{8}$$

Relationship between the discrete-time and (aperiodic continuous-time) Fourier transforms, denoted $X_d(e^{j\omega})$ and $X_c(j\omega)$, respectively:

$$X_d(e^{j\omega}) = \frac{1}{T} \sum_{k=-\infty}^{+\infty} X_c(j(\omega - 2\pi k)/T) \tag{9}$$

where T represents the sampling period. The discrete-time Fourier transform is a frequency scaled, 2π periodic version of the continuous-time Fourier transform.