



























LEMMA A.7. Let  $K_1, K_2$  be distinct copies of the domino gadget. Let  $A_1$  be a rim of  $K_1$  that is attached to a rim  $A_2$  of  $K_2$ . If  $x_{K_1}$  is inside  $A_1$ , then  $x_{K_2}$  is not inside  $A_2$ , i.e. the domino pieces can not be placed so that two adjacent squares are both black.

LEMMA A.8. Let  $K$  is a copy of the domino gadget, and let  $C_i$  be a clause gadget. Let  $A_1$  be a rim of  $K$  that is attached to a rim  $A_2$  of  $C_i$ . If  $y_i$  is inside  $A_2$ , then  $x_K$  is not inside  $A_1$ .

LEMMA A.9. If  $M$  embeds into  $\ell_\infty^2$  with distortion less than  $7/2 - O(\varepsilon)$ , then  $\varphi$  is satisfiable.

*Proof.* Considering any way of covering the domino gadgets with domino pieces and colouring at least one square of each clause gadget black. From this we will produce a satisfying assignment  $\chi$ . Consider the variable  $\chi_i$ . Because no two adjacent squares are both black, the domino pieces are placed on the rectangular path of the variable gadget  $\mathcal{V}_i$  either with each piece's black square pointing counter clockwise or each pointing clockwise. Set  $\chi_i = 1$  iff it is counter clockwise. Now consider a clause  $C_j$ . We must prove that it is satisfied by this assignment. At least one of the three squares in the clause gadget  $C_j$  must be black. Let  $\chi_i$  be the variable corresponding to the literal gadget  $\mathcal{L}_{(i,j)}$  adjacent to this black square. Because no two adjacent squares are both black, the black square of the domino pieces placed on this literal gadget must face towards the variable gadget  $\mathcal{V}_i$  ending in a black square. Hence, the square in the variable gadget  $\mathcal{V}_i$  adjacent to this literal gadget must be white. If  $\chi_i$  appears in  $C_j$  as a positive literal, then this white square is the clockwise most square in this domino gadget. Hence, we set  $\chi_i = 1$  satisfying the clause  $C_j$ . Similarly, if  $\chi_i$  appears in  $C_j$  as a negative literal.