Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

MATH1090 B. Problem Set No. 3

Posted: Nov. 5, 2021

Due: Nov. 23, 2021; by 2:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

Unless a required proof style (e.g., by resolution, Equational, Hilbert) is used in your answer, then your answer is graded out of 0.

(5 POINTS Max for each question) **Do all of the following:**

All resolution proofs below MUST use the graphical technique. Minimise preprocessing. You lose marks if your preprocessing is so long that it solves the problem WITHOUT doing any resolution step.

1. Use Resolution to prove $\vdash A \lor (B \land C) \to A \lor B$.

2. Use Resolution to prove $\vdash (A \to B) \to (A \to C) \to (A \to B \land C)$.

3. Use Resolution to prove $\vdash (p \lor q \lor r) \land (p \to p') \land (q \to p') \land (r \to p') \to p'$.

4. You are in Boolean Logic.

Prove that if a set of wff Σ is inconsistent, then it is unsatisfiable.

Note. We know that Σ is satisfiable, by definition, if some state s makes all the wff in Σ true. If no such s exists then we call Σ UNsatisfiable.

5. Prove that for any object variables $\mathbf{x}, \mathbf{y}, \mathbf{z}$ we have the absolute theorem $\vdash \mathbf{x} = \mathbf{y} \land \mathbf{y} = \mathbf{z} \to \mathbf{x} = \mathbf{z}$.

Hint. Use a Hilbert style proof using the axioms of equality. It helps if you use the (provably) equivalent form (be sure you understand what the missing, but implied, brackets say!)

$$\vdash \mathbf{x} = \mathbf{y} \to \mathbf{y} = \mathbf{z} \to \mathbf{x} = \mathbf{z}$$

6. Prove that $\vdash A \to B$ implies $\vdash (\exists \mathbf{x})A \to (\exists \mathbf{x})B$.

Hint. Use an Equational proof and the metatheorem from class "$\vdash A \to B$ implies $\vdash (\forall \mathbf{x})A \to (\forall \mathbf{x})B$".

Do NOT attempt to use the Auxiliary Hypothesis Metatheorem that we did NOT yet cover in class!

7. Prove $\vdash (\forall \mathbf{x})(A \to B) \to (\exists \mathbf{x})A \to (\exists \mathbf{x})B$.

8. Prove $\vdash (\forall \mathbf{x})(A \to (B \equiv C)) \to \Big((\forall \mathbf{x})(A \to B) \equiv (\forall \mathbf{x})(A \to C)\Big)$.