## Lassonde Faculty of Engineering EECS MATH1090. Problem Set No. 2 Posted: Oct. 4, 2017

## Due: Oct. 23, 2017, by 3:00pm; in the course assignment box.

 $\textcircled{\begin{tabular}{c}}$  It is worth remembering (from the course outline):

The homework must be each individual's <u>own work</u>. While consultations with the <u>instructor</u>, tutor, and <u>among students</u>, are part of the <u>learning</u> <u>process</u> and are encouraged, nevertheless, *at the end of all this consultation* each student will have to produce an <u>individual report</u> rather than a copy (full or partial) of somebody else's report.

"Show that —or prove that—  $\Gamma \vdash A$ " means "write a  $\Gamma$ -proof that establishes A". The proof can be Equational or Hilbert-style. Equational is rather easier in Boolean Logic.

The concept of "late assignments" does not exist in this course.

A brief but full justification of each proof step is required! Do all the following problems; (5 Points Each).

You may NOT use any of these tools: Deduction Theorem, Resolution, Post's Theorem, Cut Rule. Any solutions that use these tools will be discarded (grade 0).

- **1.** Show that  $\vdash A \rightarrow (B \rightarrow C) \equiv (A \rightarrow B) \rightarrow (A \rightarrow C)$
- **2.** Show that  $\vdash A \land B \lor A \land \neg B \equiv A$
- **3.** Show that  $A \to C \vdash A \to (B \to C)$

Limitation: Do not use problem 1!

**4.** Show that  $\vdash A \equiv B \equiv (A \land B) \lor (\neg A \land \neg B)$ 

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G. Tourlakis

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- **5.** Show that  $A \lor A \lor A \lor A \vdash B \to A$
- **6.** Suppose you are given for some formulae A and B and set  $\Gamma$  that  $\vdash A$  and  $\Gamma \vdash B$ . Show that  $\Gamma \vdash A \lor B \to A \land B$ .
- **7.** For any formula A show that  $A, \neg A \vdash \bot$

**Two separate proofs are required**: One in Hilbert style and one in Equational style.

- 8. Prove that  $\vdash A \lor B \equiv A \lor \neg B \equiv A$ .
- **9.** Prove that  $\vdash A \lor A \land B \equiv A$ .