

## Faculty of Science and Engineering

Dept. of Mathematics and Statistics

**MATH1090. Problem Set No2**

**Posted: Oct. 9, 2007**

**Due: Oct. 22, 2007; in the course assignment box.**



It is worth remembering (from the course outline):

The homework must be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, *at the end of all this consultation* each student will have to produce an individual report rather than a copy (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course.



In what follows, "prove (or "show") that  $\Gamma \vdash A$ " means that you must provide a formal (= purely syntactic) proof of  $A$  from  $\Gamma$  in either the equational or the Hilbert styles of proof. What style you choose is totally up to you but choose one that comes easily and naturally in each case.

A terse but full annotation of each proof step is required!

**Solve all the following problems (5 MARKS/Each).**

1. Suppose that  $\mathbf{q}$ , different from  $\mathbf{p}$ , does not occur in formula  $C$ . Then, by induction on  $C$  (**and only this way**), prove that the substitution  $C[\mathbf{p} := \mathbf{q}][\mathbf{q} := A]$  gives the same result as  $C[\mathbf{p} := A]$ .
2. Show that  $\vdash A \vee B \equiv A \vee \neg B \equiv A$
3. Show that  $\vdash A \wedge (A \vee B) \equiv A$
4. Show that  $\vdash A \vee A \wedge B \equiv A$
5. Show that  $\vdash A \wedge B \vee A \wedge \neg B \equiv A$
6. Show that  $\vdash A \equiv B \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$
7. Show that  $\vdash A \rightarrow (B \rightarrow C) \equiv (A \rightarrow B) \rightarrow (A \rightarrow C)$

8. Show that  $A, B \vdash A \equiv B$
9. Show that  $A, \neg A \vdash \perp$ . **Do not** use the proof in the text (which is via the “cut rule”.)