Lassonde School of Engineering

Dept. of EECS
Professor G. Tourlakis

EECS 1028 M. Problem Set No2

Posted: Feb. 4, 2023

Due: Feb. 17, 2023; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (3 MARKS) Give small examples of equivalence relations R and P such that $R \cup P$ is not an equivalence relation.

2. (3 MARKS) Given two equivalence relations R and P on A. Prove that $R \cup P$ is *reflexive* and *symmetric*.

3. (3 MARKS) Given two equivalence relations R and P on A.

Prove that $(R \cup P)^+$ is an equivalence relation.

4. (3 MARKS) Let $A \neq \emptyset$ be a set. Prove that A^2 is an equivalence relation on A.

5. (5 MARKS) Prove that for any relation R on a set A,

$$R^+ = \bigcap \Big\{ Q : R \subseteq Q \wedge \ Q \text{ is transitive} \Big\}$$

Caution. You need to prove FOUR things:

(a) The class $\Big\{ Q : R \subseteq Q \wedge \ Q \text{ is transitive} \Big\}$ is not empty. *Hint.* One of the above problems helps!

(b) $\bigcap \Big\{ Q : R \subseteq Q \wedge \ Q \text{ is transitive} \Big\}$ is a *set*. *Hint.* See whether this follows from (a) above, and if so argue succinctly why (don't write a story).

(c) $\bigcap \Big\{ Q : R \subseteq Q \wedge \ Q \text{ is transitive} \Big\}$ is transitive and $R \subseteq \bigcap \Big\{ Q : R \subseteq Q \wedge \ Q \text{ is transitive} \Big\}$

(d) If $R \subseteq S$ and S is transitive (just as in the transitive closure definition), then $\bigcap \Big\{ Q : R \subseteq Q \wedge \ Q \text{ is transitive} \Big\} \subseteq S$.

6. (4 MARKS) Let all the letters stand for integers (from $\mathbb{Z}$), with $m > 1$.

Prove that if $x \equiv y \mod m$ and $z \equiv w \mod m$, then also $x + z \equiv y + w \mod m$ and $x - z \equiv y - w \mod m$.

7. (2 MARKS) Find all *integer* values x that work in the "congruence-equation" below:

$x \equiv 8 \mod 3$.

Hint. There are infinitely many values expressible by a simple formula.