Lassonde School of Engineering

Dept. of EECS

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EECS 1028 Z. Problem Set No3

Posted: Feb. 23, 2024

Due: Mar. 22, 2024; by 6:00pm, in eClass.

Q: How do I submit?

A:

(1) **Submission must be a SINGLE *standalone* file to eClass. Submission by email is not accepted.**

(2) **Accepted File Types: PNG, JPEG, PDF, RTF, MS WORD, OPEN OFFICE, ZIP**

(3) **Deadline is strict, electronically limited.**

(4) **MAXIMUM file size = 10MB**

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. (4 MARKS) Show that if $\mathbb{F}$ is a function and $\mathrm{dom}(\mathbb{F})$ is a set then $\mathbb{F}$ is a set.

2. (3 MARKS) True or False and WHY? (without the correct "WHY" this maxes out to 0 (zero) Marks). If $\mathbb{P}$ is a function and $\mathrm{ran}(\mathbb{P})$ is a set, IS then $\mathbb{P}$ a set?

3. (3 MARKS) Prove that if the function f is 1-1, then f^{-1} —the converse of the relation f— is also a function.

Caution! The ONLY assumptions here are

1) f is a function and

2) it is 1-1.

f MAY be nontotal, non onto and have a lot of other "non" properties that you may HOWEVER NEITHER assume, NOR negate! Either way they are IRRELEVANT to the question!! **You MAY ONLY ASSUME WHAT I GAVE YOU HERE!!**

4. Given a relation $R : A \to A$. Prove

(a) (2 MARKS) $\Delta_A \circ R = R$ and

(b) (2 MARKS) $R \circ \Delta_A = R$.

5. Let $f : A \to B$ be a 1-1 correspondence. **Then Prove**:

- (3 MARKS) $f^{-1} : B \to A$ is also a 1-1 correspondence.
- (2 MARKS) If $gf = \mathbf{1}_A$, then we have $g = f^{-1}$ where f^{-1} is the converse of f.
- (2 MARKS) If $fh = \mathbf{1}_B$, then we have $h = f^{-1}$ where f^{-1} is the converse of f.

Hint. You may use relational notation if convenient, that is, "$f \circ g$" instead of "gf".

6. (4 MARKS) Let $<$ be an abstract (strict) order and $\mathbb{B}$ be any class.

Prove that $< \mid \mathbb{B}$ is an order **on** $\mathbb{B}$.

Hint. The notation "$< \mid B$" is given in the online Notes (where this Exercise is suggested for practice).

7. Suppose we know that each of A_n, $n \geq 0$, is countable.

Then do the following:

(a) (3 MARKS) Prove that $\{A_i : i \in \mathbb{N}\}$ is a set.
If you used some of the Principles 0–3 in this subquestion, be explicit!
Hint. The countability of the A_n is irrelevant to this subquestion.

(b) (4 MARKS) Prove that $\bigcup\{A_i : i \in \mathbb{N}\} = \bigcup_{i \geq 0} A_i$ is countable.

(c) (2 MARKS) Did you need the Axiom of Choice in any of the two subquestions above?
Explain WHY clearly —in a FEW words— you had to, or did not have to.

8. (a) (1 MARK) What does the name $\mathbb{V}$ stand for?

(b) (6 MARKS) Prove that the relation $\sim$ **on** $\mathbb{V}$ is *symmetric*, *transitive* and *reflexive*.