Lassonde School of Engineering

Dept. of EECS

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EECS 1028 M. Problem Set No1

Posted: Jan. 12, 2020

Due: Jan. 24, 2020; by 4:00pm, in the course assignment box.

It is worth remembering (from the course outline):

The homework **must** be each individual's own work. While consultations with the instructor, tutor, and among students, are part of the learning process and are encouraged, **nevertheless**, *at the end of all this consultation* each student will have to produce an individual report rather than a *copy* (full or partial) of somebody else's report.

The concept of "late assignments" does not exist in this course, as you recall.

1. True or False **and Why**.

(a) (2 MARKS) $\{\{1\},\{2\}\} = \{1,2\}$

(b) (2 MARKS) $\{1,1,42\} = \{42,42,1\}$

(c) (2 MARKS) $\{\emptyset\} = \emptyset$

(d) (2 MARKS) $\emptyset \subseteq \{\emptyset\}$

(e) (2 MARKS) $\emptyset \subsetneq \emptyset$

(f) (2 MARKS) $\emptyset \in \emptyset$

2. (3 MARKS) Can you find a set A that satisfies $A = \{A\}$? **Why** exactly?

3. (5 MARKS) Prove that if, for two sets A and B, we have $2^A = 2^B$, then we also have $A = B$.

Hint. Argue at the "elements level". That is, to establish $A = B$ prove for the arbitrary element/member x that we have $x \in A \equiv x \in B$.

Of course, you will prove the latter by proving *each* of $x \in A \to x \in B$ —"let $x \in A$. I will prove now $x \in B$. Etc."— **and** $x \in B \to x \in A$, as we talked about in class and posted notes.

Of course at some point you must *use* the given: $2^A = 2^B$

4. (5 MARKS) For any sets A, B, C, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

Hint. Argue at the "elements level".

5. (5 MARKS) Prove that, for any sets A and B, it is true that $A \subseteq B$ iff $A \cup B = B$.

Hint. There are two directions! lhs of iff implies rhs, and rhs of iff implies lhs.

6. Let S be a set.

(a) (3 MARKS) Is $T = \{x \in S : x \notin x\}$ a set?

(b) (3 MARKS) Can you express T **very simply** in terms of S? This is **not** a yes/no question. Either say "I do not know" or provide *with reason* such a **very simple** expression that connects T and S.

7. Use notation by explicitly listing **all the members** of each rhs $\{???\}$ to complete the following incomplete equalities:

(a) (2 MARKS) $2^{\emptyset} = \{???\}$

(b) (2 MARKS) $2^{\{\emptyset\}} = \{???\}$

(c) (2 MARKS) $2^{2^{\{\emptyset\}}} = \{???\}$

(d) (2 MARKS) $2^{2^{2^{\{\emptyset\}}}} = \{???\}$