COSC 4111/5111 —Winter 2013

Posted: March 17, 2013
Due: April 8, 2013

Problem Set No. 3

This is not a course on *formal* recursion theory. Your proofs should be informal (but $\neq$ sloppy), correct, and informative (and if possible short). Please do not trade length for correctness or readability.

(1) **Without using Rice's theorem or lemma**, explore/prove

- (a) the set $A = \{x : \mathrm{ran}(\phi_x) \text{ has exactly five distinct elements}\}$ is not recursive. (I.e., "$x \in A$ is unsolvable"). Is it r.e.? **Why?**
- (b) the set $D = \{x : \phi_x \text{ is the characteristic function of some set}\}$ is not recursive. Is it r.e.? **Why?**.
- (c) the set $E = \{x : \mathrm{ran}(\phi_x) \text{ contains only odd numbers}\}$ is not recursive. Is it r.e.? **Why?**

(2) Is the "proof" given below for the above question correct? If not, **where exactly** does it go wrong?

Proof. Let $y = f(\vec{x}_n)$ be r.e. Then $y = f(\vec{x}_n) \equiv \psi(y, \vec{x}_n) = 0$ for some $\psi \in \mathcal{P}$. Thus $g = \lambda \vec{x}_n.(\mu y)\psi(y, \vec{x}_n)$ is in $\mathcal{P}$. But $g = f$, since the unbounded search finds the y that makes $y = f(\vec{x}_n)$ true, if $f(\vec{x}_n)\downarrow$. Thus, $f \in \mathcal{P}$. $\square$

(3) Let

$$f = \lambda x.\text{if } f_R(x) = 0 \text{ then } g(x) \text{ else if } f_Q(x) = 0 \text{ then } h(x) \text{ else } \uparrow$$

where R, Q are r.e. (and mutually exclusive), and g, h, f_R, f_Q are partial recursive, and $R(x) \equiv f_R(x) = 0$ and $Q(x) \equiv f_Q(x) = 0$.

Is f partial recursive? **Why?**

Is f' below the same as f? **Why?**

$$f'(x) = \begin{cases} g(x) & \text{if } R(x) \\ h(x) & \text{if } Q(x) \\ \uparrow & \text{otherwise} \end{cases}$$

If you answered **no**, is f' partial recursive? **Why?**

(4) Do Exercise 5.2.0.32, p.358.

(5) From Section 5.3 do Problems 1, 2 and 24.