COSC 4111/5111 —Winter 2008

Posted: Jan 18, 2008
Due: TBA

Problem Set No. 1

   NB. *All problems are equally weighted out of 5.* The problem set list for grad students is the entire list here. Undergrads should omit the problems marked "Grad". If they wish to do some of those for extra credit the extra credit will be applied on an "all or nothing" basis. That is, **no part marks will be given** for a "Grad" problem attempted by undergrads.

This is not a course on *formal* recursion theory. Your proofs should be informal (but $\neq$ sloppy), correct, and informative (and if possible short). Please do not trade length for correctness or readability.

(1) Do problems 7, 11.

(2) (**Grad**). Do problem 13.

There is a typo here: "$(\mu y)_{\leq z}$" should be the "$(\overset{\circ}{\mu}\, y)_{\leq z}$" of Problem 11.

(3) Page 81, do problem 22.

(4) Prove that the projections K and L of $\lambda xy.2^{x+y+2} + 2^{y+1}$ are in $\mathcal{PR}$ but <u>do so by first finding K and L explicitly</u>.

(5) In class I claimed that $p_n \leq 2^{2^n}$ for all n. Prove this.
   Hint. Do induction. Work (as Euclid did) with $p_0 p_1 \cdots p_n + 1$.

(6) Prove that the function $\lambda x.\|x\|$, where $\|x\|$ denotes the number of binary digits of $x \in \mathbb{N}$, is in $\mathcal{PR}$.

(7) Write a "nice and clean" loop program which computes $\lambda x.\lfloor x/3 \rfloor$. The program must only allow instruction-types $X = 0$, $X = X + 1$, $X = Y$ and **Loop X ... end**. It must *not* nest the Loop-end instruction! It is required that you give a convincing general argument (*not* a "trace") as to why your program works as specified.

(8) Can loop programs

   (a) A forward **go to**? If yes, exactly how? If no, why not?

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   (b) A backward **go to**? If yes, exactly how? If no, why not?

Add to our syntax the stipulation that all instructions are labelled by numbers.

(9) (**Grad**). This requires some research; the reference is given in the problem, p.82. Your answer must be thorough and complete, not just a sketch) Do problem 25, p.82.

(10) Do problem 34, p.83.

(11) (**Grad**). Do problem 36, but change the code in (c) to $\langle 0, n, m, 2 \rangle$ throughout.