

## 1 Implementation of a dictionary with a binary search tree

### Variables

*tree*: binary tree

*invariant*: *tree* is a binary search tree; the internal nodes of *tree* contain the items of the dictionary; the external nodes of *tree* are empty

### Initialization

*tree*  $\leftarrow$  tree consisting of a single empty node

### Algorithms

**size()**:

*output*: size of dictionary

**return** (size of *tree* – 1) / 2

**isEmpty()**:

*output*: dictionary is empty?

**return** (size of *tree* = 1)

**findElement(*key*)**:

*input*: key to be searched for

*output*: element of item with *key* in dictionary; NO-SUCH-KEY if no such item exists

**return findElement(*key*, root of *tree*)**

**findElement(*key*, *node*)**

*input*: key to be searched for; root of subtree to be searched

*output*: element of item with *key* in subtree rooted at *node*; NO-SUCH-KEY if no such item exists

**if *node* is leaf then**

**return** NO-SUCH-KEY

**else if key of *node* = *key* then**

**return** element of *node*

**else if key of *node* > *key* then**

**return** findElement(*key*, left child of *node*)

**else (key of *node* < *key*)**

**return** findElement(*key*, right child of *node*)

**insertItem(*key*, *element*)**

*input*: item to be inserted

*postcondition*: item (*key*, *element*) has been inserted into dictionary

**insertItem(*key*, *element*, root of *tree*)**

**insertItem(*key*, *element*, *node*)**

*input*: item to be inserted; root of subtree to be inserted in

*postcondition*: item (*key*, *element*) has been inserted into subtree rooted at *node*

**if *node* is leaf then**

replace *node* with node containing (*key*, *element*) with two empty children

**else if key of *node*  $\geq$  *key* then**

**insertItem(*key*, *element*, left child of *node*)**

**else (key of *node* < *key*)**

**insertItem(*key*, *element*, right child of *node*)**

**remove(*key*)**:

*input*: key to be searched for

*output*: element of item with *key* in dictionary; NO-SUCH-KEY if no such item exists

*postcondition:* item has been removed from dictionary, if such an item exists

**return** remove(*key*, root of *tree*)

**remove**(*key*, *node*):

*input:* key to be searched for; root of subtree to be searched

*output:* element of item with *key* in subtree rooted at *node*; NO-SUCH-KEY if no such item exists

*postcondition:* item has been removed from subtree rooted at *node*, if such an item exists

**if** *node* is leaf **then**

**return** NO-SUCH-KEY

**else if** key of *node* > *key* **then**

**return** remove(*key*, left child of *node*)

**else if** key of *node* < *key* **then**

**return** remove(*key*, right child of *node*)

**else** (key of *node* = *key*)

*element*  $\leftarrow$  element of *node*

**if** *node* has no nonempty children **then**

replace *node* with empty leaf

**else if** *node* has only one nonempty child **then**

replace *node* with its nonempty child

**else**

*item*  $\leftarrow$  removeMin(right child of *node*)

store *item* in *node*

**return** *element*

**removeMin**(*node*):

*input:* root of subtree

*output:* item with minimal key in subtree rooted at *node*

*precondition:* *node* is nonempty

*postcondition:* node of item with minimal key has been removed from subtree rooted at *node*

**if** *node* has an empty left child **then**

*item*  $\leftarrow$  item of *node*

replace *node* with its right child

**return** *item*

**else**

**return** removeMin(left child of *node*)