Given a set of keys K, a relation R on K is a *total order* if

- for all $k \in K$, $k \, R \, k$,
- for all k_1, $k_2 \in K$, if $k_1 \, R \, k_2$ and $k_2 \, R \, k_1$ then $k_1 = k_2$,
- for all k_1, k_2, $k_3 \in K$, if $k_1 \, R \, k_2$ and $k_2 \, R \, k_3$ then $k_1 \, R \, k_3$,
- for all k_1, $k_2 \in K$, if $k_1 \, R \, k_2$ or $k_2 \, R \, k_1$.

For example,

K	R
$\mathbb{N}$	$\leq$
$\mathbb{N}$	$\geq$
words	lexicographic order

are all total orders.

PROPOSITION *Given a set of keys K and a total order R on K, there exists a key $k \in K$, such that for all $k' \in K$, $k \, R \, k'$.*
PROOF In the proof the four properties of a total order are needed. The details of the proof are out of the scope of this course. □

The key k in the above proposition is the smallest key (with respect to the total order R) of the set of keys K.