

1 Set

1.1 Specification

Set: A collection of elements (without duplicates)

size() : Returns the size of the set
isEmpty() : Tests if the set is empty
elements() : Returns the collection of elements of the set
isMember(element) : Tests if element is in the set
insertElement(element) : Adds element to the set
union(other-set) : Returns the union of the set and other-set
intersect(other-set) : Returns the intersection of the set and other-set
subtract(other-set) : Returns the set minus other-set
isEqual(other-set) : Tests if the set is the same as other-set

1.2 Implementation

Variables

set: array of elements
size: int
inv: the array set is sorted
an element occurs at most once in the array set
the array set contains the elements of the set
size is the size of the set

Initialization

size = 0

Algorithms

size()

out: size of the set
return size

isEmpty()

out: the set is empty?
return (size = 0)

elements()

out: collection of elements of the set
let col be an empty collection
for $i = 0, \dots, \text{size}-1$
 loop-inv: col contains the elements set[0], ..., set[i-1]
 add set[i] to col
return col

binarySearch(element, low, high)

pre: $0 \leq \text{low}$ and $\text{high} < \text{size}$

in: [low ... high] is the interval of the array set to be searched for element

out: is element contained in interval [low..high] of the array set
if the interval [low ... high] is empty
return false

else

set middle to be the middle of the interval [low ... high]

if set[middle] = element

return true

else if set[middle] > element

return binarySearch(element, low, middle-1)

else

return binarySearch(element, middle+1, high)

isMember(element)

in: element to be searched for

out: element in the set?

return binarySearch(element, 0, size-1)

insertElement(element)

in: element to be added to the set

post: element has been added to the set

if element is not a member of the set

$i = \text{size} - 1$

while set[i] > element and $i \geq 0$

loop-inv: set[$i + 1$], ... set[size-1] are moved one position to the right in the array set

move set[i] one position to the right

decrement i

set[$i + 1$] = element

increment size

union(other-set)

in: set to be added

out: union of the set and other-set

let temp be an array of elements

$h = 0$

$i = 0$

$j = 0$

while $i < \text{size}$ and $j < \text{size of other-set}$

loop-inv: {temp[0], ..., temp[$h - 1$]} = {set[0], ..., set[$i - 1$]} \cup {other-set[0], ..., other-set[$j - 1$]}

if set[i] = other-set[j]

temp[h] = set[i]

increment h , i and j

else if set[i] < other-set[j]

temp[h] = set[i]

increment h and i

else

temp[h] = other-set[j]

increment h and j

while $i < \text{size}$

loop-inv: {temp[0], ..., temp[$h - 1$]} = {set[0], ..., set[$i - 1$]} \cup {other-set[0], ..., other-set[$j - 1$]}

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    temp[h] = set[i]
    increment h and i
while j < size of other-set
    loop-inv: {temp[0], ..., temp[h - 1]} = {set[0], ..., set[size - 1]} ∪
        {other-set[0], ..., other-set[j - 1]}
    temp[h] = other-set[j]
    increment h and j
return (temp, h)

intersect(other-set)
in: set to be intersected with
out: intersection of the set and other-set
let temp be an array of elements
h = 0
i = 0
j = 0
while i < size and j < size of other-set
    loop-inv: {temp[0], ..., temp[h - 1]} = {set[0], ..., set[i - 1]} ∩ {other-set[0], ..., other-set[j - 1]}
    if set[i] = other-set[j]
        temp[h] = set[i]
        increment h, i and j
    else if set[i] < other-set[j]
        increment i
    else
        increment j
return (temp, h)

subtract(other-set)
in: set to be subtracted
out: subtraction of other-set from the set
let temp be an array of elements
h = 0
i = 0
j = 0
while i < size and j < size of other-set
    loop-inv: {temp[0], ..., temp[h - 1]} = {set[0], ..., set[i - 1]} \ {other-set[0], ..., other-set[j - 1]}
    if set[i] = other-set[j]
        increment i and j
    else if set[i] < other-set[j]
        temp[h] = set[i]
        increment h and i
    else
        increment j
while i < size
    loop-inv: {temp[0], ..., temp[h - 1]} = {set[0], ..., set[i - 1]} \ {other-set[0], ..., other-set[j - 1]}
    temp[h] = set[i]
    increment h and i
return (temp, h)

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isEqual(other-set)
in: set to be checked to be equal to the set
out: set is equal to other-set?
i = 0
equal = (size = size of other-set)
while i < size and equal
    loop-inv. equal = (size = size of other-set and {set[0], ..., set[i]} = {other-set[0], ..., other-set[i]}
    if set[i] ≠ other-set[i]
        equal = false
    else
        increment i
return equal
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