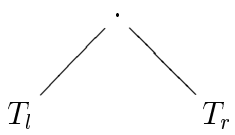
PROPOSITION 7.2 *Let T be an AVL tree with n nodes and height h.*

1. $n > 2^{\frac{h}{2}-1}$
2. $h < 2\log(n) + 2$

PROOF

1. We prove this proposition by induction on n. The base cases $n = 1$ (and hence $h = 0$) and $n = 2$ (and hence $h = 1$) are easy. Next we consider the induction step. Let $n > 2$. Assume that the above claim is true for all nonempty AVL trees with less than n nodes. The AVL tree T is of the form



   where the number n_l of nodes of the left subtree T_l and the number n_r of nodes of the right subtree T_r are both smaller than n. Clearly, T_l and T_r are nonempty AVL trees. Since T has the height-balance property, we have that

   - T_l and T_r both have height $h - 1$, or
   - T_l has height $h - 1$ and T_r has height $h - 2$, or
   - T_l has height $h - 2$ and T_r has height $h - 1$.

   We only consider the last case. The others can be proved similarly.

$$\begin{aligned} n &= n_l + n_r + 1 \\ &> 2^{\frac{h-2}{2}-1} + 2^{\frac{h-1}{2}-1} + 1 \quad \text{[induction]} \\ &> 2 \cdot 2^{\frac{h-2}{2}-1} \\ &= 2^{\frac{h-2}{2}} \\ &= 2^{\frac{h}{2}-1}. \end{aligned}$$

2. By 1., $n > 2^{\frac{h}{2}-1}$. Hence, $\log(n) > \frac{h}{2} - 1$. So, $h < 2\log(n) + 2$.

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