Definition 1 A *cut* C is a finite subset of events such that if $e \in C$ and e and e' are events of the same process and $e' < e$, then $e' \in C$.

Definition 2 The *global time* of a cut C is

$$\tau(C)[i] = |\{ e \in C \mid e \text{ is an event of process } P_i \}|.$$

Observation 1 $C_1 \subseteq C_2$ if and only if $\tau(C_1) \leq \tau(C_2)$.

Definition 3 The *causal past* of an event e is

$$\downarrow e = \{ e' \mid e' < e \text{ or } e' = e \}.$$

Definition 4 A cut C is *consistent* if $e \in C$ and $e' < e$ then $e' \in C$.

Observation 2 $\downarrow e$ is a consistent cut.

Observation 3 Let C be a consistent cut. Then

$$e \in C \text{ if and only if } \downarrow e \subseteq C.$$

Observation 4 Let $\mathcal{C}(e)$ be the vector time of event e. Then

$$\mathcal{C}(e)[i] = |\{ e' \mid e' \text{ is an event of process } P_i \text{ and } e' < e \text{ or } e' = e \}|.$$

Observation 5 $\tau(\downarrow e) = \mathcal{C}(e)$.

Observation 6 If $e \neq e'$ then $\mathcal{C}(e) \neq \mathcal{C}(e')$.

Correctness and completeness result $e < e'$ if and only if $\mathcal{C}(e) < \mathcal{C}(e')$.
Proof

$e < e'$

$\iff$ $e \in \downarrow e'$ and $e \neq e'$

$\iff$ $\downarrow e \subseteq \downarrow e'$ and $e \neq e'$ [Observation 2 and 3]

$\iff$ $\tau(\downarrow e) \leq \tau(\downarrow e')$ and $e \neq e'$ [Observation 1]

$\iff$ $\mathcal{C}(e) \leq \mathcal{C}(e')$ and $e \neq e'$ [Observation 5]

$\iff$ $\mathcal{C}(e) < \mathcal{C}(e')$ [Observation 6]