Quantitative Bisimulation for Probabilistic Processes: a counterexample (note)

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In [GJS90], Giacalone, Jou and Smolka introduce a quantitative notion of bisimulation for deterministic probabilistic processes. In the conclusion of their paper, they suggest a generalization for arbitrary (i.e. nondeterministic probabilistic) processes. In the presence of only one label, their definition boils down to the following

Definition 1 An equivalence relation $\mathcal{R}$ on processes is an *ϵ-bisimulation* if $P \mathrel{\mathcal{R}} Q$ implies that for all $\mathcal{R}$-equivalence classes S,

* if $P \xrightarrow{\kappa} S$ then $Q \xrightarrow{\lambda} S$ for some λ such that $|\kappa - \lambda| \leq \epsilon$, and
* if $Q \xrightarrow{\kappa} S$ then $P \xrightarrow{\lambda} S$ for some λ such that $|\kappa - \lambda| \leq \epsilon$.

Processes P and Q are *ϵ-bisimilar*, written $P \overset{\epsilon}{\sim} Q$, if $P \mathrel{\mathcal{R}} Q$ for some ϵ-bisimulation $\mathcal{R}$. ⌟

This leads to a natural notion of distance between processes.

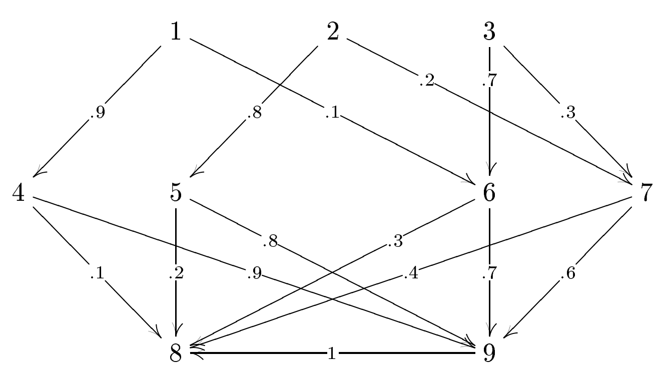
Definition 2 The distance function d on processes is given by

$$d(P,Q) = \begin{cases} \inf \{ \epsilon \mid P \overset{\epsilon}{\sim} Q \} & \text{if } P \overset{\epsilon}{\sim} Q \text{ for some } \epsilon \\ 1 & \text{otherwise.} \end{cases}$$

⌟

Giacalone, Jou and Smolka mention that this distance function does not satisfy the triangle inequality. Here, we given a counterexample.

Example 1 Consider the following processes and their transitions.



One can easily verify that the smallest equivalence relations containing $\{\langle 1,2\rangle, \langle 4,5\rangle, \langle 6,7\rangle\}$ and $\{\langle 2,3\rangle, \langle 5,6\rangle\}$ are .1-bisimulations. Therefore, $1 \overset{.1}{\sim} 2$ and $2 \overset{.1}{\sim} 3$. Hence, $d(1,2) \leq .1$ and $d(2,3) \leq .1$. If the triangle inequality would hold for d, then $d(1,3) \leq .2$. However, we will show next that there exists no ϵ-bisimulation $\mathcal{R}$, with $\epsilon \leq .25$, such that $1\,\mathcal{R}\,3$. This is proved in the following three steps.

1. Let $\mathcal{R}$ be an ϵ-bisimulation, with $\epsilon \leq .25$. Let S be the $\mathcal{R}$-equivalence class with $8 \in S$. Since $8 \xrightarrow{0} S$ and $9 \xrightarrow{1} S$, we can conclude that $8 \not\mathcal{R} 9$.

2. Let $\mathcal{R}$ be an ϵ-bisimulation, with $\epsilon \leq .25$. Let S be the $\mathcal{R}$-equivalence class with $8 \in S$. According to 1, $9 \notin S$. Because $4 \xrightarrow{.1} S$ and $7 \xrightarrow{.4} S$, we can deduce that $4 \not\mathcal{R} 7$.

3. Let $\mathcal{R}$ be an ϵ-bisimulation, with $\epsilon \leq .25$. Towards a contradiction, assume that $1\,\mathcal{R}\,3$. Let S be the $\mathcal{R}$-equivalence class with $4 \in S$. Clearly, $6 \in S$. Let T be the $\mathcal{R}$-equivalence class with $7 \in T$. Then $4 \in T$ or $6 \in T$. Hence, 4 and 7 are in the same $\mathcal{R}$-equivalence class, which contradicts 2.

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References

[GJS90] A. Giacalone, C.-C. Jou, and S.A. Smolka. Algebraic Reasoning for Probabilistic Concurrent Systems. In *Proceedings of the IFIP WG 2.2/2.3 Working Conference on Programming Concepts and Methods*, Sea of Gallilee, April 1990. North-Holland.