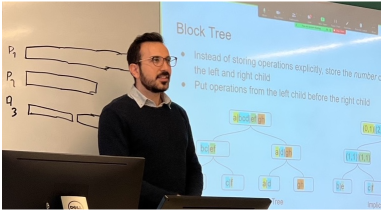
A Wait-Free Queue with Polylogarithmic Step Complexity

Hossein Naderibeni   Eric Ruppert

June 21, 2023



Queues: Breaking Linear-Time Bottleneck

Problem: implement linearizable, lock-free FIFO queue

- shared by p processes
- use single-word CAS (reasonable-sized words)
- support multiple enqueuers, dequeuers

Many previous solutions for this problem.

All require $\Omega(p)$ steps per operation
$\rightarrow$ Real obstacle to scalability

Our New Queue

- $O(\log p)$ steps per ENQUEUE
- $O(\log^2 p + \log q)$ steps per DEQUEUE ($q =$ size of queue)
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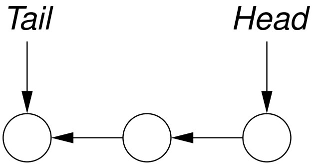
Lock-Free Queue using CAS

Michael and Scott Queue [PODC 1996]



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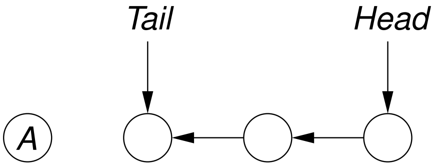

ENQUEUE(*A*):

1. create new node

Lock-Free Queue using CAS

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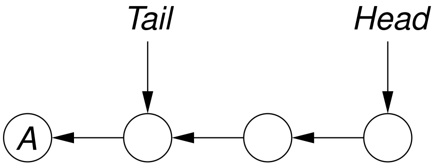

ENQUEUE(A):

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2. CAS next pointer

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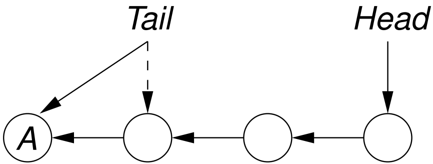

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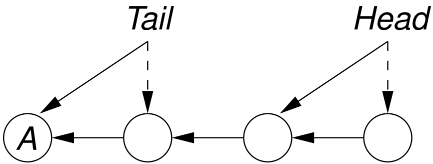

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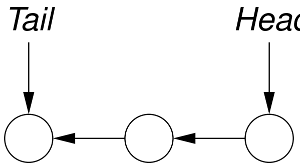
DEQUEUE:

1. CAS *Head*

CAS Retry Problem

Suppose p processes want to enqueue simultaneously.



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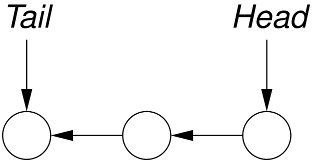

ENQUEUE(*A*) (*A*)

ENQUEUE(*B*) (*B*)

ENQUEUE(*C*) (*C*)

ENQUEUE(*D*) (*D*)

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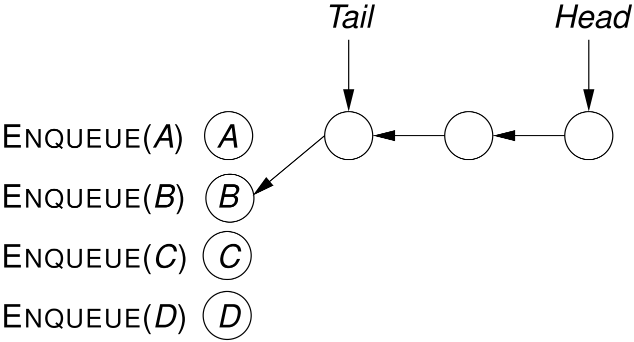

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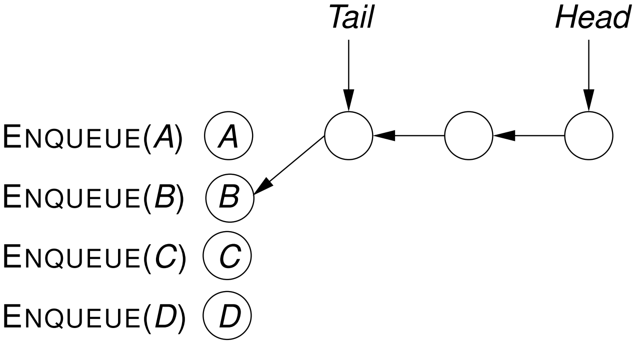

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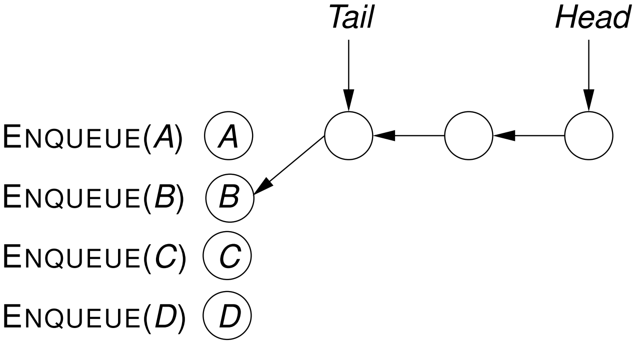

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Other Lock-Free Queues

Other list-based queues

- add elimination array [Moir et al. 2005]
- baskets queue [Hoffman, Shalev, Shavit 2007]
- doubly-linked list + optimism [Ladan-Mozes, Shavit 2008]
- fast path, slow path [Kogan, Petrank 2012]
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So do array-based queues

All* previous queues take amortized $\Omega(p)$ steps per operation

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*Exceptions: Sublinear Time Queues

Restricted queues

- 1 enqueuer, multiple dequeuers [David 2004]
- 1 dequeuer, multiple enqueuers [Jayanti, Petrovic 2005]

Other primitives

- $O(\sqrt{p})$ using unusual double-word RMW instructions [Khanchandani, Wattenhofer 2018]

Universal constructions

- $O(\log p)$ using huge words [Afek, Dauber, Touitou 1995; Jayanti 1998]
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Lower Bound

All previous multi-enqueuer, multi-dequeuer queues take $\Omega(p)$ steps per operation.

For many data structures, fastest lock-free operations take $O(\textit{sequential complexity} + \textit{contention})$ steps

Lower Bound [Attiya, Fouren 2017]

- Amortized step complexity for any bag is $\Omega(\textit{contention})$.
- But lower bound holds only if *contention* is $O(\log \log p)$

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Breaking Linear-Time Bottleneck

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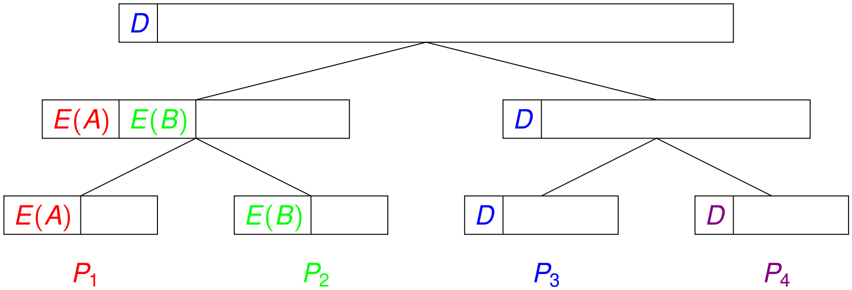
- $O(\log^2 p + \log q)$ steps per DEQUEUE
- wait-free
- uses CAS on reasonable-size words
- bounded space version: $O(\log p \log (p + q))$ amortized steps per operation (relies on safe GC)

$p = \#$ processes
$q = \#$ elements in queue

Ordering Tree



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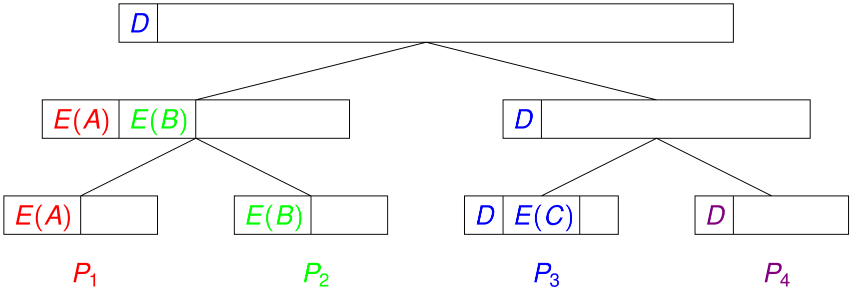

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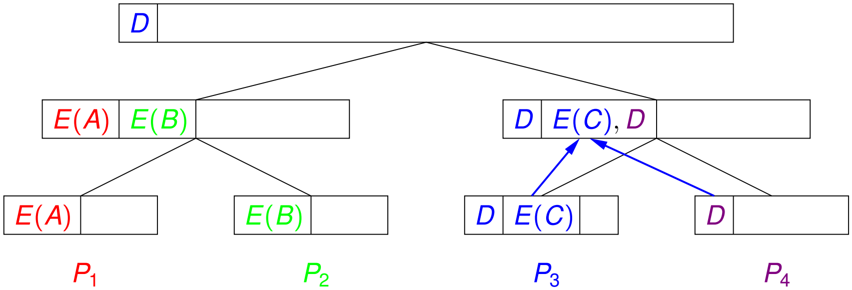

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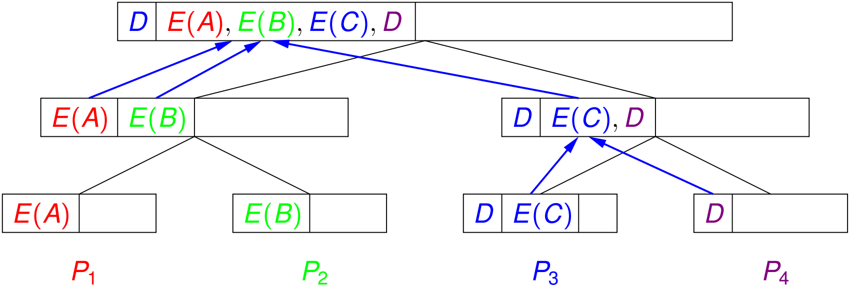
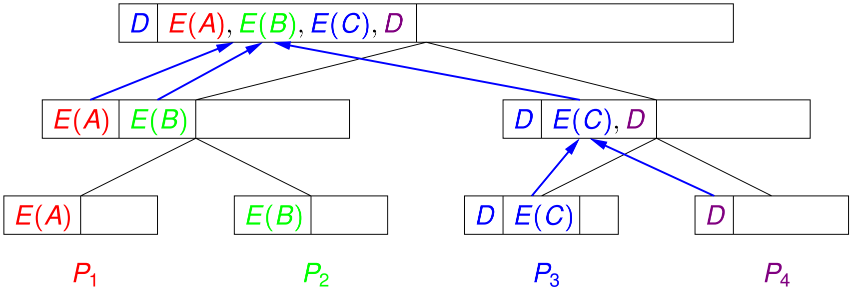


Ordering Tree

Use ordering in root as linearization



Propagating Operations to the Root

1. Append operation to your leaf
2. At each node v on path to root refresh *twice*:
   (a) Read unpropagated operations in both of v's children
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Double Refresh

If your CAS on v fails twice, then
another process has propagated your operation to v.

Avoids CAS retry problem.

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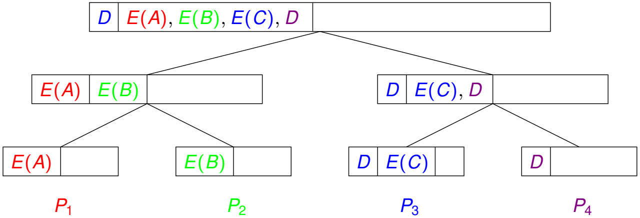
Main Challenge

Refresh may have to propagate up to p operations
$\Rightarrow$ need an implicit representation

Requirements for Implicit Representation



Must support following in polylog time

- Refresh: promote batch of ops from children to parent
- Find my DEQUEUE in root
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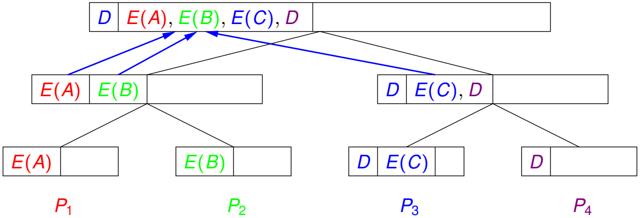

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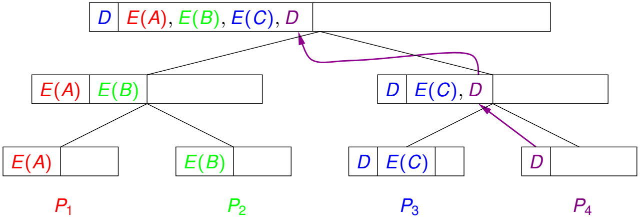

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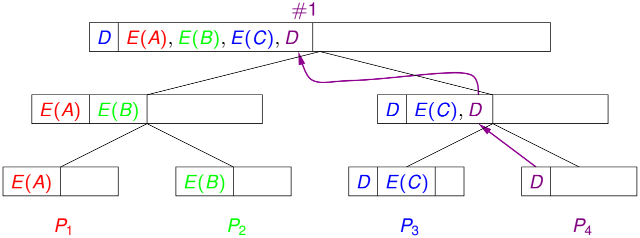

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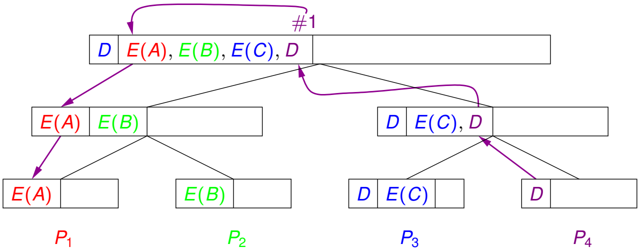

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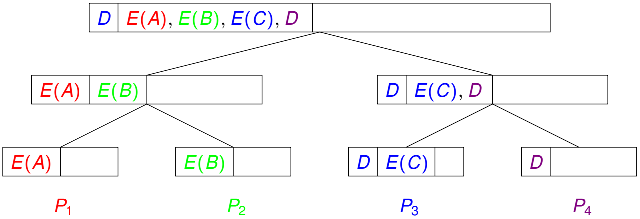

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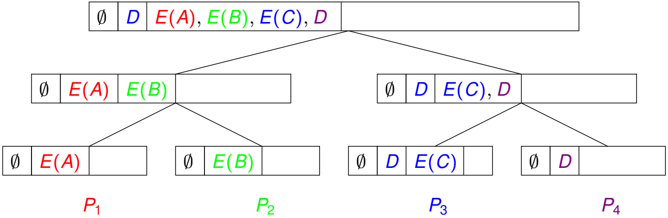

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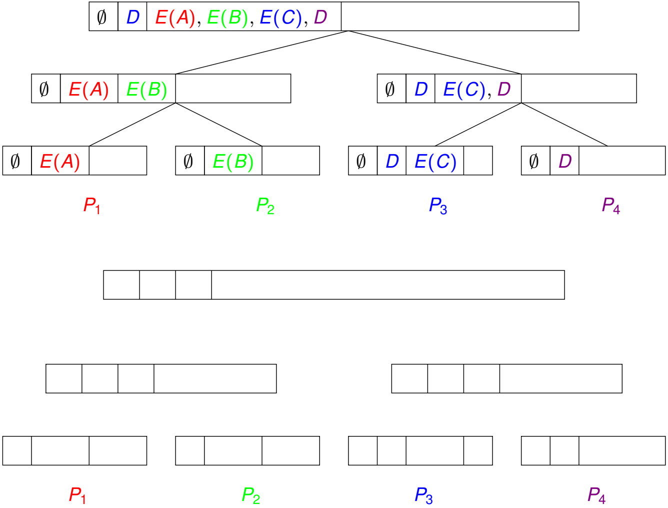

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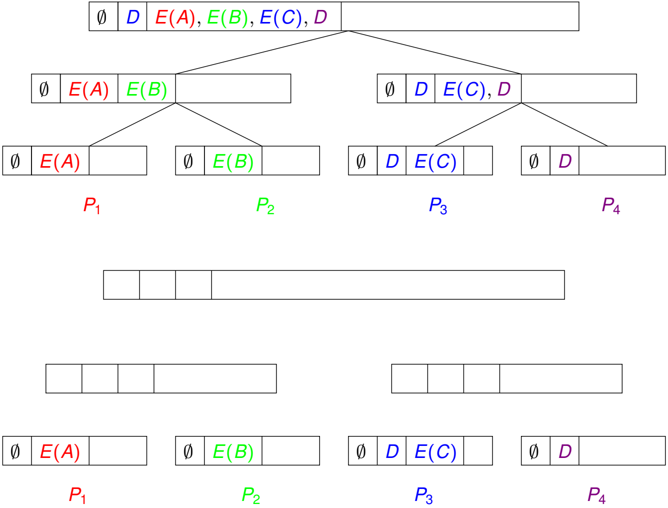

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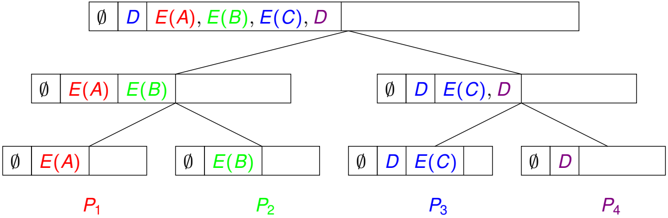

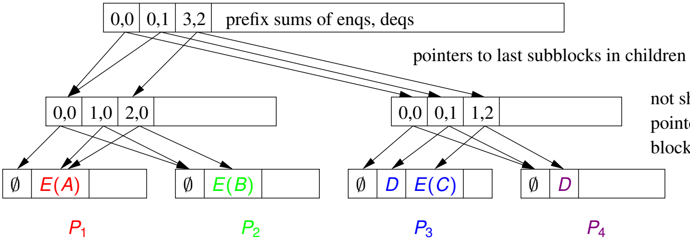

not shown: approximate pointer from subblock to block in parent

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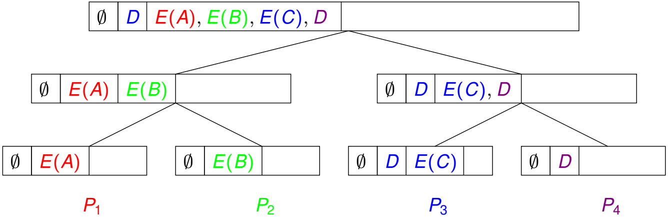
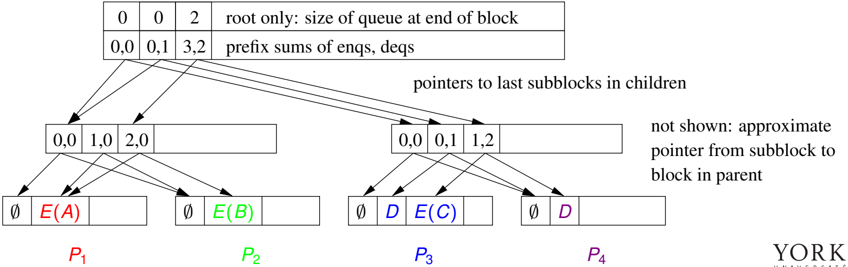
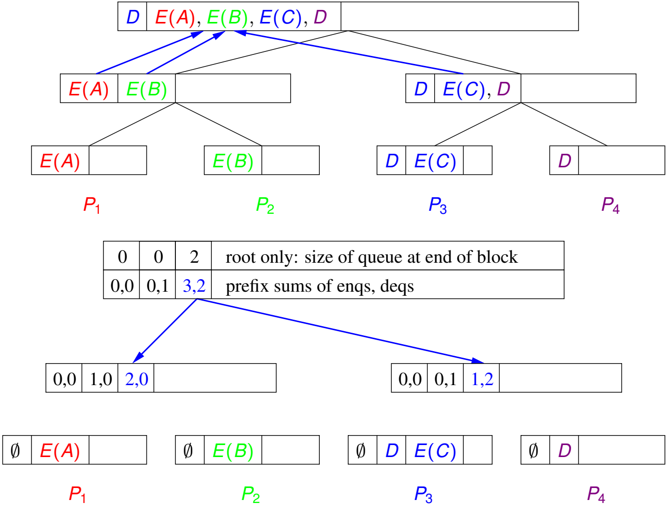
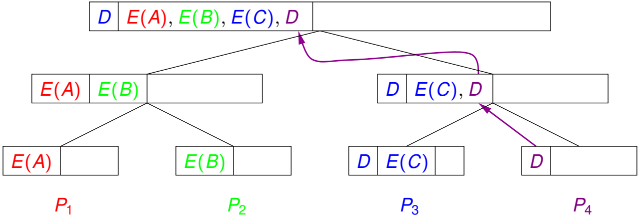


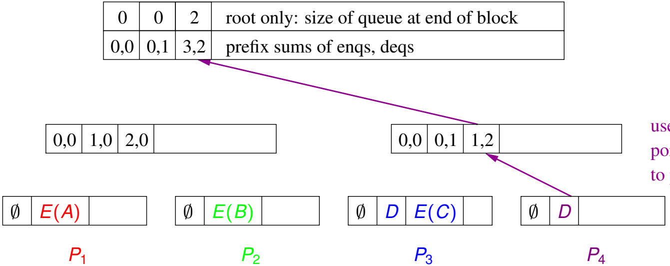


Adding a Block for a Refresh



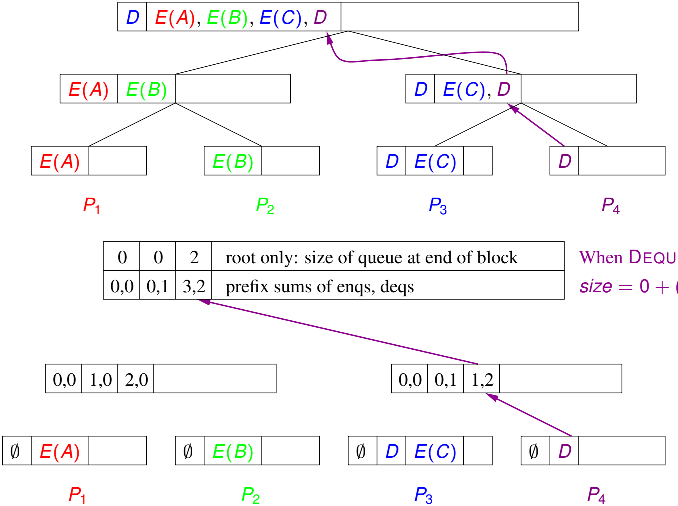
Tracing a DEQUEUE to the Root





use approximate
pointers from sublocks
to parent's block

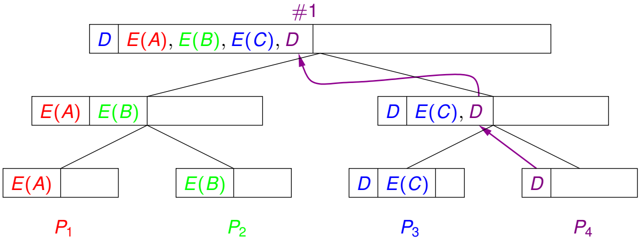
Check if DEQUEUE Returns Null



When DEQUEUE occurs,

$size = 0 + (3 - 0) > 0$

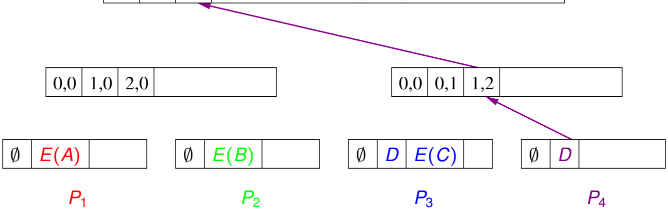
Rank of DEQUEUE Among Non-Null DEQUEUES



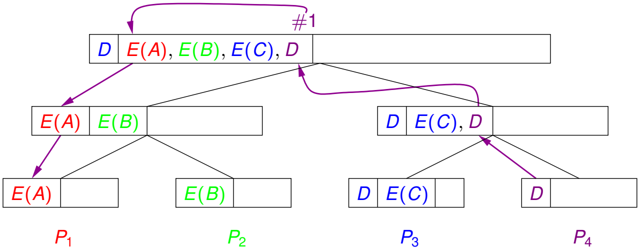
0	0	2	root only: size of queue at end of block
0,0	0,1	3,2	prefix sums of enqs, deqs

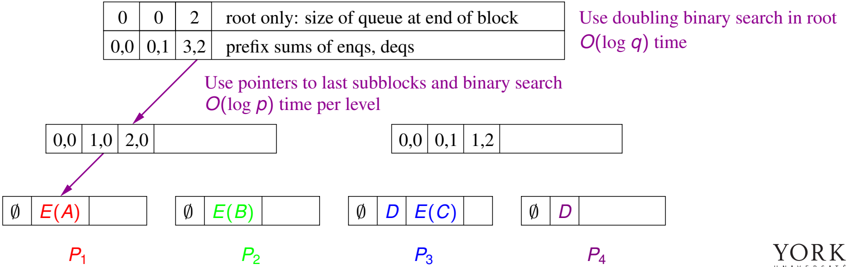
use *size* = #enqs − # non-null deqs

⇒ #non-null deqs = #enqs − *size*



Find ENQUEUE of Given Rank





Our Result

New Wait-Free Queue

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- $O(\log p)$ CAS steps per DEQUEUE
- Unbounded space

$p = \#$ processes
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Bounding Space

- Replace each array of blocks with a red-black tree of blocks
- Periodically split RBT and discard obsolete blocks
- Processes help one another to ensure blocks are obsolete

Bounded-Space Queue

- Amortized $O(\log p \log (p+q))$ steps per operation
- $O(pq + p^3 \log p)$ space
- Still wait-free

$p = \#$ processes
$q = \#$ elements in queue

Future Directions

- Practical implementation
  (perhaps slow path of fast path slow path method)
- Extend technique to other data structures
  (stacks and deques are recently done)
- Close gap between
  $\Omega(\log \log p)$ lower bound [Attiya Fouren 2017]
  $O(\log^2 p + \log q)$ upper bound [this work]