York University COSC6117 March 26, 2002

Homework Exercise
Due: April 8, 2002

9. The following programme is a randomized self-stabilizing leader election algorithm that uses one single-writer snapshot object. Each component of the snapshot object stores a single bit. If the bit is 1, it means that the corresponding process thinks he's a leader. The goal is to ensure that, for any initial state, eventually, there will be a 1 in exactly one component of the snapshot object and 0's in all the other components (with probability 1). Assume that in any execution, every process takes infinitely many steps (i.e. no process dies).

Process P_i executes the following code:

loop forever
  $x[1..n] \leftarrow$ result of a scan of snapshot object
  if (every element of x is 0) OR ($x[i] = 1$ and $x[j] = 1$ for some $j \neq i$) then
    update component i of the snapshot object to 0 or 1, chosen randomly (each value
      having probability $\frac{1}{2}$)
  end if
end loop

Consider an execution α that starts from an arbitrary configuration. Assume α is long enough so that every process does a complete iteration of the loop. Furthermore, assume that, for each update operation, the value that is chosen in α for the update is 1 iff all other components of the snapshot object are 0 at the moment just before the update occurs. (In other words, α is an execution where we get really lucky and all the randomly chosen values turn out to be just what we would wish them to be.)

(a) Show that once there is a leader at some point in α, there will always be at least one leader afterwards.

(b) Show that there is eventually a leader in α.

(c) Show that at the end of α, there is a unique leader.

(d) Show that the probability of all the random choices having the outcomes described above is at least 2^{-n}. Hint: what's the maximum number of updates a process can perform in α?

(e) A round is a segment of an execution where every process takes at least one step. Explain why the expected number of rounds before there is a unique leader is $O(2^n)$, regardless of the initial state or the order in which processes take their steps.