Homework Assignment #3
Due: April 30, 2001

1. [10 marks] Recall that a monotone circuit is a circuit that contains no NOT gates. A monotone alternating circuit is a monotone circuit where no AND gate has another AND gate as its input, and no OR gate has another OR gate as its input. (Thus, if you follow any path through a monotone alternating circuit, you will alternately hit AND and OR gates.) For the MONOTONE-ALTERNATING-CIRCUIT-VALUE-PROBLEM (MACVP), you are given a monotone alternating circuit with no variable gates, and you must decide whether the output gate evaluates to TRUE. Show that MACVP is **P**-complete.

2. [5 marks] For the PLANAR-CLIQUE problem, you are given a *planar* graph, and you must determine whether it contains a clique of size K (i.e. a set of K vertices, between any two of which there is an edge). Is PLANAR-CLIQUE in **P** or is it **NP**-complete? Prove your answer. You may use the following fact without proving it: A planar graph with n vertices has fewer than $3n$ edges.

3. [10 marks] Show that 2-COLOURABILITY is in **P**.

4. [10 marks] Consider a version of SAT where every clause contains exactly three literals, and no variable appears twice in the same clause. Show that this version of SAT is **NP**-complete.

5. [10 marks] In the SPLITTING problem, you are given a finite set U and n subsets of U: $S_1, S_2, \ldots, S_n$. You must decide whether it is possible to divide U into two subsets U_1 and U_2 such that $U = U_1 \cup U_2$ and no set S_i is entirely contained in either U_1 or U_2. Show that SPLITTING is **NP**-complete by giving a reduction from 3SAT. Hint: One possible reduction is similar to (though simpler than) the reduction from 3SAT to 3-COLOURABILITY given in class.

6. [10 marks] Show that the INDEPENDENT SET problem is still **NP**-complete if the input graphs do not contain any triangles (i.e. cliques of three nodes). Hint: Try replacing each edge of a graph by a little graph so that the resulting graph contains no triangles and the size of the resulting graph's largest independent set is related to the size of the original graph's largest independent set.