York University COSC6115 February 28, 2001

Homework Assignment #1
Due: March 21, 2001

1. [10 marks] Give a Turing machine that subtracts one from a number given in binary. For example, if the input is 10101000, the machine should halt with output 10100111, and if the input is 10000, the machine should halt with output 01111.

2. [10 marks] In the textbook definition of a multi-tape Turing machine, each cursor can either move left one square, right one square or remain stationary in each step. A "multi-tape Turing machine with reset" is one where, in each step, each cursor can either move right one square, remain stationary or jump all the way to the left end of the tape. That is (using the notation of Definition 2.4), δ maps $K \times \Sigma^k$ to $(K \cup \{h, \text{"yes"}, \text{"no"}\}) \times (\Sigma \times \{\text{RESET}, \rightarrow, -\})^k$. Note that a cursor cannot move just one square to the left, as it can in an ordinary Turing machine. Explain how a given multi-tape Turing machine can be simulated by a multi-tape Turing machine with reset, with a polynomial increase in the number of steps. (It's possible to do the simulation with a quadratic increase in the number of steps.) Hint: One way to solve this problem is to store, on auxiliary tapes, binary counters that keep track of the cursor positions on the tapes of the original machine.

3. [10 marks] This question requires you to prove some closure properties of complexity classes.

[5] **(a)** Show that $\mathbf{TIME}(f(n))$ is closed under union. That is, show that if L_1 and L_2 are in $\mathbf{TIME}(f(n))$, then so is $L_1 \cup L_2$.

[5] **(b)** Let L be a language. Define L^* to be $\{w_1 w_2 \ldots w_k : k \geq 0 \text{ and } w_i \in L \text{ for all } i\}$. Show that if $L \in \mathbf{NTIME}(n)$ then $L^* \in \mathbf{NTIME}(n)$.

4. [10 marks] Let P be the set of strings of properly nested and matched parentheses. For example, (()(()()))(()()) and (()((()))) are in P, but ()) and (()))()(() are not in P. Show that $P \in \mathbf{L}$ by giving an informal description of a Turing machine that decides the language P using $O(\log n)$ space.

5. [10 marks] There is a mistake in Theorem 2.5.

[3] **(a)** Give a counterexample to the theorem for the case where $f(n) = O(1)$. That is, give a function ϕ and show that ϕ can be computed by a RAM programme in $O(1)$ time but cannot be computed by a Turing machine in $O(1)$ time.

[3] **(b)** Point out where there is an error in the proof of Theorem 2.5.

[4] **(c)** Fix the statement of the theorem, and explain how to modify the proof so that it is correct.