Breast Cancer Detection using Time Reversal Signal Processing

Mohammad Sajjadih, Foroohar Foroozan, and Amir Asif
Department of Computer Science and Engineering
York University, Toronto, Canada M3J 1P3

Abstract—Breast cancer is the second leading cause of cancer death after lung cancer among women. The value of advanced imaging platforms such as magnetic resonance imaging (MRI) for early breast cancer detection remains unclear. The paper applies a time reversal beamforming imager [15] for detecting and locating early stage breast cancer tumours from MRI data. In our simulations based on the finite difference, time domain (FDTD) electromagnetic model, the proposed detector estimates the locations of breast cancer tumours with a higher accuracy than some of the current state of art signal processing estimation algorithms tested by us.

I. INTRODUCTION

Breast cancer is the second leading cause of cancer death after lung cancer among women affecting one in every seven women [1] in North America. Each year more than 200,000 new cases of invasive breast cancer are diagnosed and more than 40,000 women die from the disease in the US and Canada alone [2]. For years, X-ray mammography [3] was the only practical procedure for detection of breast cancer. However, diagnostic mammography frequently generates many abnormal findings leading to additional, costly imaging procedures and biopsies. This is especially true for young women, who present a higher ratio of dense to fatty tissues in their breast, limiting the effectiveness of X-ray mammography in such cases.

In the last decade, research [4]–[6] has shown that microwave breast cancer detection has the potential to become a successful clinical complement to conventional X-ray mammography. Microwave imaging utilizes non-ionizing electromagnetic radiation to detect cancer tumours in a human body by exploiting differences between the dielectric properties of normal and malignant tissues. Active microwave imaging techniques for breast cancer detection may broadly be classified into two different categories: (i) tomographic methods, and; (ii) backscatter methods. The basic difference between the two approaches is that the tomographic methods are based on measurements of the microwave radiation transmitted through the breast, while backscatter methods use reflections from the tumour tissues within the breast. The tomographic approaches are based on the work of Greenleaf et al. [7], who made the seminal observation that acoustic measurements made with transmission ultrasound could be used to characterize breast tissue. On the basis of these studies, they concluded that using the parameters of sound speed

and attenuation could help differentiate benign masses from cancer. Using a number of in-vitro samples of various types of breast tissue, they were able to quantify the sound speed (propagation velocity of an ultrasound pulse passing through the tissue) and attenuation (the reduction in pulse amplitude as it propagates through the tissue). They also demonstrated that, in a plot of sound-speed as a function of attenuation, the benign and malignant masses were well separated. As a direct result of this and other similar studies, a number of investigators [7]–[10] developed transmission ultrasound scanners in an attempt to measure transmission parameters and to replicate their results in vivo. The problem with microwave tomography, however, is the computationally complexity of the algorithm used to solve the ill-conditioned nonlinear inverse scattering problem of determining the dielectric profile for the breast tissue from sound speed and attenuation measurements. A second challenge is its high vulnerability to noise from observation and experimental uncertainties. Another challenge is the accurate modeling of the antenna array configuration used for signal transmission/detection in tomographic devices, which can adversely affect the accuracy in identifying the location of the cancerous tumour.

In backscatter imaging, several microwave emitters illuminate the breast and the resulting backscatter (i.e., reflection) is measured at multiple detectors. Malignant breast tumours have electrical properties that are significantly different from those of healthy breast tissues [8],[9]. For example, the cancerous tumour produces a stronger backscattered electromagnetic energy return as compared to the reflections from the normal tissues. In principle, it is then possible to locate the tumours from the backscatters Fear et al. [6],[10] demonstrate the feasibility of detecting and localizing small tumours of less than 1 cm diameter with numerical simulations involving synthetic cylindrical and planar antennas. Stoica et al. [13] use ultra-wideband confocal microwave imaging (CMI) [12] to achieve higher resolution and suppress interference. However, unlike X-rays, which are non-diffractive and travel in straight lines, electromagnetic microwave propagation in breast tissues is characterized by refraction and multipath effects, i.e., the backscattered cancer signature signal reaches the detector by two or more paths. As a result, standard signal processing algorithms do not perform well due to multipath propagation and cannot accurately identify nor locate cancer tumours with sufficiently fine resolution. Below, we discuss a different backscatter imaging paradigm based on time reversal signal

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processing that uses multipath propagation to its advantage for breast cancer detection.

A. Time Reversal

Time reversal, matched field processing (TR/MFP) is an emerging technology that has received considerable attention in acoustics. Though the exact schematics of the TR/MFP configuration depend upon the application in consideration, but, in general, most existing TR/MFP techniques exploit the phenomena of super-resolution focusing observed by following the steps outlined below.

- 1) An active target or scatterer, embedded in an unknown medium, generates a signature pulse $f(t)$ into the random medium. If the target is passive, then a transducer array illuminates the unknown medium with a probing pulse such that the reflection from the target constitutes the signature pulse $f(t)$.
- 2) The transducer array, placed either within or outside the medium, records the waveforms at each of its transducer elements. For passive targets, as is the case for breast tumours, the backscatters of the probing pulse form the observations.
- 3) Each transducer element time reverses its recorded waveform and retransmits the time-reversed wave into the medium in conjunction with other elements at the same time.
- 4) The retransmitted waves focus strongly at the location of the target. In the context of TR/MFP, this phenomena is referred to as the super resolution focusing.

In physical TR, Steps 3 and 4 are performed in real, while in computational TR, Steps 3 and 4 are performed via computer simulations. TR/MFP offers an alternative to traditional detectors, e.g. the matched filter in one dimensional signal processing and the matched field processing (MFP) in multidimensions. The difficulty with MFP is that the Green's function used to model the channel is not known and, therefore, has to be computed numerically. By its inherent nature, TR/MFP intrinsically derives the Green's function of the channel as long as the receiving transducer array provides an adequate sampling of the channel. A second advantage of TR/MFP lies in the productive treatment of the channel multipaths. As is generally known, multipath affects the performance of traditional detectors significantly and is considered to be a detrimental and a negative whose effects should be minimized. TR/MFP presents the opposite opportunity: multipath as a positive, the more the better. Though the technique of TR/MFP is not new, but a thorough theory of detection for this setting is lacking. Our initial work [11] and [12] addresses this gap and proves *analytically*, we believe for the first time, the phenomena of super resolution focusing observed with TR/MFP. We have also developed TR imaging algorithms [13]-[14] to detect and accurately estimate the location of a target in a high scattering environment with strong clutters. In this paper, we extend the TR based imaging algorithms [15] to breast cancer detection. Our hypothesis is that TR based beamforming imaging exploits successfully the multiple path

Fig. 1. Simulation model for the breast with tumour shown as a sphere ($\bullet$) of 1mm diameter.

EM wave scattering due to the inhomogeneous breast tissues to detect and diagnose small cancer tumours with high sensitivity and high specificity. We have validated this hypothesis through initial simulations based on numerical finite difference time domain (FDTD) modeling. In our simulations, our proposed TR-based approach outperforms the current state of art in breast cancer detection identifying the location of tumour with a higher precision that what was previously thought possible.

The organization of the paper is as follows. Section II describes the proposed TR breast detection algorithm followed by a report in Section III on our continuing work showing the results we have obtained with an anatomically realistic FDTD simulation model based on an MRI scan of a human breast. Finally, Section IV concludes the paper.

II. TR BREAST DETECTION IMAGING ALGORITHMS

Our TR cancer detection (TRCD) algorithm is based on the setup shown in Fig. 1, where Array A probes the breast using the wideband signal $f(t)$, $0 \leq t \leq T$, with the discrete time Fourier transform (DTFT)

$$\forall 0 \leq q \leq (Q-1), \quad \omega_q = \omega_0 + q\Delta\omega, \quad (1)$$

while Array B records the backscatter of the probing signal during the forward propagation step. In Eq. 1, ω_0 is the lowest frequency present in the signal and the frequency step $\Delta\omega$ equals $2\pi/(Q-1)T$. Our algorithm is similar to the one proposed by Moura et al. [15]. The TRCD algorithm consists of five steps.

Step 1 (Clutter Channel Probing): The TRCD algorithm consists of a training stage, where the breast is probed in the absence of any tumour to estimate the clutter response matrix $\mathbf{K}_c(\omega_q)$. In reality, patients' chronicle EM screening data or other imaging modalities may be used to obtain the estimate of strong reflections from the clutter (patients skin in this case). The probing transceiver array transmits the narrowband signal $F(\omega_0)$, such that each probing antenna n , ($1 \leq n \leq N$), radiates signal $F(\omega_0)$, one element at a time. The scattered

returns are received at the probing antenna array, $p = 1, \dots, P$. The probing step is repeated L times at frequency ω_0 to average out the observation noise, $1 \leq l \leq L$. When element n is transmitting, the received signal vector $\mathbf{r}_n(\omega_0)$ of the l 'th observation is given by

$$\mathbf{r}_{n,l}(\omega_0) = \mathbf{K}_c(\omega_0)\mathbf{e}_n F(\omega_0) + \mathbf{n}_l(\omega_0), \quad (2)$$

where $\mathbf{e}_n$ is the vector whose n 'th entry is 1 and the rest is 0 and $\mathbf{n}_l(\omega_0)$ is the additive noise vector. From these L measurements, the clutter channel frequency response at frequency ω_0 is estimated. For white Gaussian noise with $F(\omega_0) = 1$, Eq. (2) simplifies to

$$\hat{\mathbf{K}}_c(\omega_0)\mathbf{e}_n = \frac{1}{L} \sum_{l=1}^L \mathbf{r}_{n,l}(\omega_0). \quad (3)$$

Eq. (3) is used to derive the n^{th} column in $\mathbf{K}_c(\omega_0)$. The process is repeated for all of the antenna elements n , for $1 \leq n \leq N$, to form $\mathbf{K}_c(\omega_0)$. Step 1 is also repeated for frequencies, ω_q , $0 \leq q \leq (Q-1)$, such that the clutter response $\mathbf{K}_c(\omega_q)$ is derived for all frequencies.

Step 2 (Clutter Cancellation through Beamforming): Assume element n of the probing array is transmitting, the signals recorded at the observing array are all time-reversed and retransmitted back into the medium. The retransmitted signals will focus at the location of the clutter. In TRCD, the recorded signals are time-reversed but before transmitting, they are reshaped with a $(P \times P)$ matrix $\mathbf{W}(\omega_q)$ in order to nullify the effect of clutters. At discrete frequency ω_q , ($0 \leq q \leq Q-1$), when the n 'th element in the probing array is transmitting the signal $F(\omega_q)$, the P signals recorded at the observing array is $\mathbf{K}_c(\omega_q)F(\omega_q)\mathbf{e}_n$. After TR and filtering with $\mathbf{W}(\omega_q)$, the resulting signals are given by

$$\mathbf{x}_n(\omega_q) = \mathbf{W}(\omega_q)\mathbf{K}_c^*(\omega_q)F^*(\omega_q)\mathbf{e}_n, \quad (4)$$

for ($1 \leq n \leq N$) and ($0 \leq q \leq Q-1$). The clutter backscattered signal received at the probing array is given by

$$\mathbf{y}_n(\omega_q) = \mathbf{K}_c^T(\omega_q)\mathbf{x}_n(\omega_q), \quad (5)$$

where subscript n denotes that element n has probed the channel initially. The process is repeated for all frequencies ω_q and all antenna elements n of the probing array such that the final observation is given by

$$\mathbf{y} = [\mathbf{y}(\omega_0), \dots, \mathbf{y}(\omega_{Q-1})]^T, \quad (6)$$

where $\mathbf{y}(\omega_q) = [\mathbf{y}_1(\omega_q), \dots, \mathbf{y}_N(\omega_q)]^T$, for ($0 \leq q \leq Q-1$), and T denotes transposition. In terms of the clutter response matrix, reference [15] shows that the filter $\mathbf{W}(\omega_q) = k_q(\mathbf{K}_c^*(\omega_q)\mathbf{K}_c^T(\omega_q))^{-1}$, which nullifies the effect of the clutter.

Step 3 (Monitoring for Breast Tumours): Assuming that the tumour is present, the signal received at the antenna array when element n probes the channel is given by

$$\begin{aligned} \mathbf{z}'_n(\omega_q) &= (\mathbf{K}_t(\omega_q) + \mathbf{K}_c(\omega_q))^T \mathbf{x}_n(\omega_q) \\ &= \underbrace{k_q \mathbf{K}_t^T(\omega_q) \mathbf{K}_c^{-T}(\omega_q) \mathbf{e}_n F^*(\omega_q)}_{\mathbf{z}_n^c(\omega_q)} + \underbrace{k_q \mathbf{e}_n F^*(\omega_q)}_{\mathbf{z}_n^t(\omega_q)} \end{aligned} \quad (7)$$

where $\mathbf{K}_t(\omega_q)$ is the target (tumour) response matrix and $\mathbf{z}_n^t(\omega_q)$ the tumour's response. If the known clutter component $\mathbf{z}_n^c(\omega_q)$ is subtracted out and tumour response is stacked, we have

$$\begin{aligned} \mathbf{Z}(\omega_q) &= [\mathbf{z}_1^t(\omega_q), \dots, \mathbf{z}_N^t(\omega_q)] \\ &= k_q \mathbf{K}_t^T(\omega_q) \mathbf{K}_c^{-T}(\omega_q) F^*(\omega_q). \end{aligned} \quad (8)$$

Step 4 (TR Detection): After the whitened clutter has been subtracted out, returns from step 3 are either from noise or target response plus noise. The probing array time reverses the signal in Eq. (8) and retransmits it again to obtain focused returns from the target. The signal plus clutter components of the received signal at the observing array are given by

$$\begin{aligned} \mathbf{p}'_n(\omega_q) &= (\mathbf{K}_t(\omega_q) + \mathbf{K}_c(\omega_q))^T [\mathbf{z}_n(\omega_q)]^* \\ &= k_q \mathbf{K}_t(\omega_q) \mathbf{K}_t^H(\omega_q) \mathbf{K}_c^{-H}(\omega_q) \mathbf{e}_n F(\omega_q) \\ &\quad + k_q \mathbf{K}_c(\omega_q) \mathbf{K}_t^H(\omega_q) \mathbf{K}_c^{-H}(\omega_q) \mathbf{e}_n F(\omega_q) \\ &= \mathbf{p}_n^t(\omega_q) + \mathbf{p}_n^c(\omega_q). \end{aligned} \quad (9)$$

Again, the clutter component is subtracted to get $\mathbf{p}_n^t(\omega_q)$ for $n = 1, \dots, N$. Combining all the vectors $\mathbf{p}_n^t(\omega_q)$ for all probing elements, gives an $P \times P$ matrix $\mathbf{M}^p(\omega_q)$ as follows:

$$\begin{aligned} \mathbf{M}^p(\omega_q) &= [\mathbf{p}_1^t(\omega_q), \dots, \mathbf{p}_N^t(\omega_q)] \\ &= k_q \mathbf{K}_t(\omega_q) \mathbf{K}_t^H(\omega_q) \mathbf{K}_c^{-H}(\omega_q) F(\omega_q). \end{aligned} \quad (10)$$

Steps 3 and 4 are repeated but in the opposite order such that we start initially from the observing array instead of the probing array. The clutter target response matrix measured at the probing array is presented as the $N \times N$ matrix $\mathbf{M}^n(\omega_q)$ as follows:

$$\mathbf{M}^n(\omega_q) = k_q \mathbf{K}_t^T(\omega_q) \mathbf{K}_t^*(\omega_q) \mathbf{K}_c^{-*}(\omega_q) F(\omega_q) \quad (11)$$

Step 5 (TR Image Generation): This step forms the image by scanning the area under interest with two focused beams, one at the observing array and the other at the probing array. Notations $\mathbf{w}_{rp}(\mathbf{x}, \omega_q)$, $\mathbf{w}_{tp}(\mathbf{x}, \omega_q)$, $\mathbf{w}_{rn}(\mathbf{x}, \omega_q)$, and $\mathbf{w}_{tn}(\mathbf{x}, \omega_q)$ denote the receive and transmit weight functions for the observing and probing antenna arrays respectively at frequency ω_q , $\forall q$. The complex output of the beamformers at the observing and probing arrays are given by

$$\mathbf{Y}^p(\mathbf{x}, \omega_q) = \mathbf{w}_{rp}^H(\mathbf{x}, \omega_q) \mathbf{M}^p(\omega_q) \mathbf{w}_{tp}(\mathbf{x}, \omega_q) \quad (12)$$

$$\text{and } \mathbf{Y}^n(\mathbf{x}, \omega_q) = \mathbf{w}_{rn}^H(\mathbf{x}, \omega_q) \mathbf{M}^n(\omega_q) \mathbf{w}_{tn}(\mathbf{x}, \omega_q), \quad (13)$$

where $\mathbf{x}$ is the location of each pixel in the region of interest (ROI), which is used in imaging. In order to form the final image $\mathbf{I}(\mathbf{x})$ at each pixel $\mathbf{x}$, we multiply the outputs of the two beamformers at each frequency and use the spatial distribution of the total energy as given by

$$\mathbf{I}(\mathbf{x}) = \sum_{q=0}^{Q-1} |\mathbf{Y}^n(\mathbf{x}, \omega_q) \mathbf{Y}^p(\mathbf{x}, \omega_q)|^2. \quad (14)$$

Eq. (14) implements the *energy detector*. The optimal weights in Eqs. (12) and (13) are calculated from the Green's functions vectors and clutter multistatic matrix. In the next section, we apply the TRCD algorithm for breast cancer detection.

III. EXPERIMENTAL RESULTS

In order to quantify the performance of the proposed TRCD algorithm, we run a two dimensional (2D) finite difference, time domain (FDTD) simulation operating in the transverse magnetic (TM) mode. The electromagnetic (EM) properties of the breast needed to run the simulation are derived from the actual MRI's of real patients. An example of a test MRI is shown in Fig. 2(a), while Fig. 2(b) shows the experimental setup obtained from the test MRI used in our simulations. The thick line (shown in light green) in Fig. 2(b) models the skin, which has a thickness of 5mm, conductivity of $\sigma = 0.4\text{S/m}$ and permittivity of $\epsilon_r = 30$. The breast is assumed to be immersed in a lossless liquid so the region outside the breast has conductivity $\sigma = 0\text{S/m}$ and permittivity $\epsilon_r = 9$. Both arrays A and B contain 12 elements each and are located in the lossless liquid, as shown, respectively, by 'x' and '*' in Fig. 2(b). To obtain permittivity ϵ_r for the breast tissues, we map the average intensity values of the MRI to the average value for the permittivity observed in breast tissues using a linear transformation. In the MRI we tested, the average intensity value of the pixels is 210. Based on our database, the average permittivity of a normal breast tissue is 9. This implies that pixel values be divided by the factor of $210/9$, or, roughly 24 to obtain the permittivity values for the breast tissues. Based on this transformation, the breast tissues has a variability of 20% in the permittivity values. This is very much in line with the variability of (16%-24%) observed in the actual MRI database [16]. The conductivity of the model is also achieved by a similar linear transformation. Based on our database, the average conductivity of the normal breast tissue is $\sigma = 0.4\text{S/m}$. Therefore, the divider used to convert the the intensity values of a breast MRI to the conductivity values is $210/0.4$, or, 525. Since the chest is not projected in Fig. 2(b), we add it manually to our breast model. The chest wall includes muscles and ribs and is shown at the bottom of Fig. 2(b). To know the ground reality against which the performance of the algorithm will be tested, the tumour is inserted manually within the normal breast MRI before deriving the EM properties of the breast. The location of the tumour, selected at random, is shown as a bright region close to coordinates (10, 15.5) cm in Fig. 2(b). Within the tumour, the maximum values for $\epsilon_r = 50$ and $\sigma = 4\text{S/m}$.

FDTD based electromagnetic simulations are run on the 2D domain specified in Fig. 2(b) to compare the performance of the proposed TRCD algorithm with the current state-of-art detection algorithms used in medical signal processing. The algorithms that we tested include: (a) the Multiple Signal Classification (MUSIC) algorithm; (b) MUSIC algorithm coupled with TR; (c) Maximum likelihood (ML) estimator, and; (d) the proposed TRCD algorithm. In all four algorithms, direct subtraction is used to derive the target response multistatic matrix K_t from the clutter and clutter/target response matrices. Generally, MUSIC require the number of targets and scatterers to be less than the number of array elements. Because in our breast model, distinct scatterers are more than the antenna

(a)

Fig. 2. (a) MRI of a normal breast. (b) EM model derived from plot (a) to test the TRCD algorithm using FDTD simulations.

elements, MUSIC produces poor results. Therefore, we also use a combination of TR and MUSIC (algorithm b) and a combination of ML and MUSIC (algorithm c). The detection results are shown in Fig. 3, where $\circ$ represents the centeroid of the actual tumour and $\times$ represent the estimated location. Comparing the results of four algorithms shows that the estimate of the proposed TRCD algorithm is closest to the actual location of the tumour. Note that the tumour is located close to the chest wall and far from both antenna arrays. This is a challenging situation because reflections from tumour and the chest wall combine and also because significant secondary reflections exist between the chest wall and tumour. The filtering step, used in the TRCD algorithm, mitigates reflections from clutters including from the skin and chest wall. The proposed algorithm, therefore, provides the best result.

IV. SUMMARY

The paper applies the time reversal beamforming imager [15] for detecting breast cancer tumours from MRI data. We test the performance of the proposed algorithm against three state-of-art detection algorithms used in medical signal processing. In our simulations based on the FDTD electromagnetic model, the proposed detector estimates the location of the tumour with a much higher accuracy.

Fig. 3. Experimental results illustrating the superiority of the proposed TRCD algorithm over its counterparts. Outputs of: (a) the MUSIC algorithm; (b) the MUSIC algorithm coupled with TR; (c) the ML estimator, and; (d) the proposed TRCD algorithm. Symbol $\circ$ represents the centroid of the tumour and $\times$ represent the estimated location. Estimate of the TRCD algorithm is the most accurate.

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