Photoresponse of CMOS Image Sensors

by

Ji Soo Lee

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2003

© Ji Soo Lee 2003
I hereby declare that I am the sole author of this thesis.

I authorize the University of Waterloo to lend this thesis to other institutions or individuals for the purpose of scholarly research.

Signature

I further authorize the University of Waterloo to reproduce this thesis by photocopying or by other means, in total or in part, at the request of other institutions or individuals for the purpose of scholarly research.

Signature
Borrower’s Page

The University of Waterloo requires the signatures of all persons using or photocopying this thesis. Please sign below, and give address and date.
Acknowledgements

I would like to thank Dr. Richard I. Hornsey for the unwavering support, direction, guidance, and encouragements that he has provided throughout this work. The gracious and intelligent manner by which he supervised my post-graduate study is also gratefully appreciated and acknowledged.

The fabrication and design support from Canadian Microelectronics Corporation has enabled the experiments carried out in this work. The unwavering help from Jim Quinn at Canadian Microelectronics Corporation is especially noted.

Special thanks to my colleagues at VISOR Laboratory at York University for all the memorable and enriching experiences shared over the past several years.

Finally, to my parents, my deep thanks and gratitude.
Abstract

An improved one-dimensional analysis of CMOS photodiode quantum efficiency has been derived in which the effect of the substrate, which forms a high-low junction with the epitaxial layer, has been included. The analytical solution was verified with numerical simulations based on parameters extracted from a standard 0.35 µm CMOS process. Two empirical parameters are suggested to offset the unavoidable inaccuracies in the extracted parameter values. The derived semi-empirical expression exhibits a good agreement with the measured spectral response.

The lateral photoresponse in CMOS photodiode arrays is investigated with test linear photodiode arrays and numerical device simulations. It is shown that the surface recombination and mobility degradation along the Si-SiO₂ interface are important factors in determining the lateral photoresponse of CMOS photodiodes. The limitations of traditional analytical approaches are briefly discussed in this context, and a novel three-dimensional analysis of lateral photoresponse is presented. Given the significant dependence of lateral photoresponse on the Si-SiO₂ interface quality, an empirical characterization method is proposed as a more reliable solution to modeling lateral photoresponse.

Lateral crosstalk in CMOS imaging array deter effective utilization of small pixel sizes (e.g. < 5.0 µm x 5.0 µm) now permitted by technology scaling. A relatively simple measurement technique for an empirical characterization of lateral crosstalk is presented. A demonstration of deblurring operations based on the obtained blur model of lateral crosstalk is also presented. The tradeoffs in sharpness restoration and high-frequency noise amplification in the design of linear deblurring operation for the application of lateral crosstalk are illustrated.
Index Terms—CMOS photodiode, CMOS image sensor, quantum efficiency, active pixel sensor (APS), lateral diffusion, lateral photoresponse, edge-effect, lateral crosstalk.
Table of Contents

Borrower’s Page ....................................................................................................................................... iii
Acknowledgements ......................................................................................................................................... iv
Abstract ......................................................................................................................................................... v
Table of Contents .......................................................................................................................................... vii
List of Figures .................................................................................................................................................. ix
List of Tables ................................................................................................................................................ xii

Chapter 1 Introduction ................................................................................................................................. 1
1.1 Silicon Photodetectors ......................................................................................................................... 3
1.2 CMOS Photodiodes ............................................................................................................................... 3
1.3 CMOS Passive and Active Pixel Sensors ............................................................................................ 5
1.4 Integrating and Non-Integrating Image sensors ................................................................................... 6
1.5 Fixed Pattern Noise ............................................................................................................................... 8
1.6 Double Sampling Techniques ............................................................................................................. 8
1.7 Dark Current .......................................................................................................................................... 9
1.8 Temporal Noise ..................................................................................................................................... 9
1.9 Lateral Diffusion and Lateral Crosstalk ............................................................................................... 10
1.10 Motivation and Organization of Thesis .............................................................................................. 11

Chapter 2 Quantum Efficiency .................................................................................................................... 14
2.1 Survey of Photodiode Analyses .......................................................................................................... 14
2.2 One-Dimensional Analysis ................................................................................................................... 15
List of Figures

Fig. 1.1 Energy band of silicon ...............................................................................................................3
Fig. 1.2 Absorption distance (1/α) in silicon as a function of wavelength ............................4
Fig. 1.3 CMOS compatible photodiode structures ..................................................................................5
Fig. 1.4 CMOS passive and active pixel sensors ..................................................................................6
Fig. 1.5 Architectural view of integrating type image sensor with frame-scan readout ..........7
Fig. 1.6 Architectural view of non-integrating type image sensor with random access readout ....8
Fig. 1.7 An illustration of lateral diffusion of photocarrier in an imaging array .........................10
Fig. 2.1 Measured external quantum efficiency of n⁺-p_epi photodiode .........................................17
Fig. 2.2 Two images of CMOS epitaxial layer taken with a scanning electron microscope ......19
Fig. 2.3 Measured external quantum efficiency of n⁺-p_epi photodiode .........................................20
Fig. 2.4 Simple one-dimensional description of n⁺-p_epi type photodiode ....................................20
Fig. 2.5 Excess minority carrier density in the substrate as a function of the substrate to epitaxial
doping ratio ...........................................................................................................................................25
Fig. 2.6 Comparison of the derived expression of photocurrent and numerical simulation ........26
Fig. 2.7 Semi-empirical quantum efficiency fitted by varying substrate diffusion length ..........28
Fig. 2.8 Final expression of quantum efficiency with empirical parameters .............................30
Fig. 3.1 Micrograph of the fabricated linear photodiode array .......................................................34
Fig. 3.2 Measured photocurrent from the linear photodiode arrays ...............................................35
Fig. 3.3 Measured lateral photoresponse versus numerical simulation ........................................36
Fig. 3.4 The top and cross sectional views of two-dimensional rectangular photodiode array ....39
Fig. 3.5  Comparison of the measurement and the derived lateral photocurrent expressions......45
Fig. 3.6  Measured edge-effect characterized by E_{2D}...........................................................................47
Fig. 3.7  Device simulation of two-dimensional array........................................................................50
Fig. 4.1  A simple linear model of CMOS image sensor........................................................................54
Fig. 4.2  The measurement setup for cross-responsivity PSF...............................................................54
Fig. 4.3  Examples of CMOS active pixel sensors layout.......................................................................56
Fig. 4.4  Measured cross-responsivity PSF obtained from four CMOS image sensors..................57
Fig. 4.5  A test image consisting of several black stripes on a white background convolved with
   the cross-responsivity PSF of IBIS4.................................................................................................59
Fig. 4.6  Inverse filtering on captured images of IBIS4..........................................................................61
Fig. 4.7  An illustration of excessive high-frequency gain in a deblurring operation.........................62
Fig. 4.8  Photoresponse non-uniformity measurements on IBIS4 and ZR32112.........................63
Fig. 4.9  Comparison of inverse filtering, Wiener deconvolution, and CLSR.................................66
Fig. 4.10 Comparison of inverse filtering with two 3x3 local sharpening operators....................68
Fig. 4.11 Illustration of the results in Table 4.2...................................................................................70
Fig. A.1  An example of one-dimensional photodiode structure.......................................................77
Fig. C.1   Schematic of photodiode-type active pixel sensor ..............................................................90
Fig. C.2   Illustration of photodiodes with one or more substrate openings......................................91
Fig. C.3  An illustration of the reduction in junction capacitance with a single square-shaped
   substrate opening....................................................................................................................92
Fig. C.4  Micrographs taken from some of fabricated photodiodes................................................93
Fig. C.5  Normalized photocurrent versus area of single square-shaped substrate opening...........94
Fig. C.6  Measured photocurrent divided by the photodiode junction capacitance.......................95
Fig. C.7 Normalized photocurrent versus the sum areas of multiple substrate opening ...............96

Fig. C.8 Measured photocurrent divided by the photodiode junction capacitance ....................97

Fig. C.9 Normalized photocurrent versus sum area of 72 instances of 1 µm x 1 µm substrate
openings..........................................................................................................................98
List of Tables

Table 4.1  Some Information about the four CMOS image sensors in Fig. 4.4.............................57
Table 4.2  Improvement of geometric MTF with pixel size reduction..........................................70
Chapter 1

Introduction

Charge-couple devices (CCDs) were invented by S. Boyle and G. G. Amelio at Bell Laboratory in 1970, and it has become the dominant technology for solid-state image sensor. While CCDs offer low noise, low non-uniformity, and low dark current in relation to their present CMOS image sensor counterparts, they also have several drawbacks, including complex clocking requirements, high power consumption, difficulty of on-chip integration of circuitry, and limited frame rate [1]. CMOS image sensors address these drawbacks by using the same technology as microprocessors and memory chips – CMOS (Complementary Metal Oxide Semiconductor) technology. The choice leads to image sensors with all of the advantages of the latest fabrication technology:

- **Low Power Consumption**: CMOS image sensors draw significantly less power than CCDs, which lends them to be more suitable for battery-powered portable applications such as laptop computers, digital cameras, video cell-phones, and hand-held scanners [1].

- **System Integration**: Signal conditioning circuits, microprocessors, memory, and buses can all be integrated onto a single-chip, which makes CMOS image sensors easier to produce and more cost-effective than CCDs. The on-chip integration of standard camera functions reduces the part-count of the imaging system, which in turn boosts reliability, eases miniaturization. Modern single-chip CMOS image sensors offer on-chip programming of frame size, windowing, exposure, and other camera parameters [2].

- **Speed**: The on-chip integration of all necessary components in CMOS image sensors significantly increase the rate of data transfer between the sensor and the processing units,
which in turn translates to faster frame rate. CMOS image sensors with 10,000 frames per second have been reported [3].

With these advantages CMOS image sensors are emerging as a complementary solution to CCDs. Their present applications include internet camera, digital still camera, machine vision, automotive, children’s toy, medicine and dentistry, fingerprint ID, surveillance, aerospace, motion analysis, industrial inspection, quality control, process control, target tracking, and spectroscopy [4], [5].

1.1 Silicon Photodetectors

A photodetector converts incident radiation into photocurrent that is generally proportional to the radiant power. The incident photons excite electron-hole pairs in the semiconductor when the photon energy ($E_{ph}$) is greater than the band-gap energy of the semiconductor ($E_g$). The light induced free carriers are referred to as photocarriers. Photodetectors collect photocarriers and convert them into a photocurrent. Since the photocurrent is generally very small, it is integrated into charge and then converted to voltage in photovoltaic mode of operation [6].

Consider the energy band diagram of silicon in Fig. 1.1. An incident photon requires at least 1.124 eV of energy to excite an electron from the valence band to the conduction band. The photons in the visible range have enough energy to generate electron-hole pairs in the silicon: at 400nm (violet) $E_{ph} = 3.1$ eV, at 700nm (red) $E_{ph} = 1.77$ eV, and at 1100 nm (infrared) $E_{ph} = 1.12$ eV.
Fig. 1.1 Energy band of silicon

\[ E_g = 1.124 \text{ eV} \]

Fig. 1.2 Absorption depth \((1/\alpha)\) in silicon as a function of wavelength [8].
The rate of photocarrier generation below the incident surface of a semiconductor is given by [7]

\[ G(x) = \alpha F_0 e^{-\alpha x} \]  

(1.1)

where \( F_0 \) is the incident photon flux (photons/cm\(^2\)) and \( \alpha \) is the absorption coefficient of the semiconductor medium (cm\(^{-1}\)). Fig. 1.2 provides a plot of the absorption coefficient for silicon crystal. It follows that 99% of violet and blue light are absorbed within depth of approximately \( x \approx 0.6 \) \( \mu \)m and red light require depth of \( x \approx 16.6 \) \( \mu \)m for the same.

### 1.2 CMOS Photodiodes

Three different photodiodes can be implemented with a standard CMOS technology as shown in Fig. 1.3. The essential difference between the different structures of photodiode is the location and the size of the photodiode depletion region, which constitutes the photodiode’s collection region. A shallow junction photodiode is preferable for short-wavelength illuminations as the majority of the photons are absorbed relatively close to the illuminated surface, while a photodiode whose junction extends deep into the epitaxial layer is more suitable for long-wavelengths that penetrate deeper into the silicon. One can maximize the quantum efficiency of a photodiode by extending the photodiode depletion region from the (illuminated) surface of the semiconductor to the depth at which the incident radiation trails off. Since any increase in the photodiode depletion region increases the photodiode’s dark current, the depletion region should be constrained to regions of optical excitation. In many pixel architectures, the internal capacitance of a photodiode is also important to the pixel operation, as illustrated in the following
sections. Therefore, an optimum photodiode structure can significantly vary depending on the application of the photodiode, which may dictate the target wavelength (e.g. infrared imaging), relative importance of dark current, and other aspects of photodiode that may be significant to the pixel operation.

Fig. 1.3 CMOS compatible photodiode structures. (a) n⁺-pₑ₊, (b) p⁺-nwell, and (c) nwell-pₑ₊ photodiodes [9].

### 1.3 CMOS Passive and Active Pixel Sensors

A passive pixel sensor (PPS) generally consists of a photodetector and a pass transistor. The photodetector is connected to a vertical column bus (Fig. 1.4). The column bus is kept at a constant voltage by a charge-integrating amplifier [10]. The photodetector generally operates in photovoltaic mode in PPS where the photocurrent is integrated into a charge for a period of time. When the photodetector is accessed, the accumulated charge is transferred to the charge-integrating amplifier, and the voltage of the photodetector is reset to the column bus voltage. The charge-integrating amplifier converts the transferred charge into a voltage output. The period of time between the photodetector reset and the readout operation during
which the photocarrier charge accumulates in photodetector is referred to as the integration time.

![CMOS passive and active pixel sensors](image)

Fig. 1.4 CMOS passive and active pixel sensors.

An active pixel sensor (APS) has an amplifier in each pixel (Fig. 1.4). The in-pixel amplifier is commonly implemented with a single transistor in source-follower configuration. The in-pixel amplifier in APS enables non-destructive read of the photodiode charge at a faster speed and a generally higher signal-to-noise ratio (SNR) than PPS [5]. A disadvantage of APS is lower fill factor resulting from the additional transistor(s) required to implement the in-pixel amplifier. Since technology scaling has enabled significant reduction in transistor sizes, APS is the pixel architecture of choice in modern CMOS image sensors.

### 1.4 Integrating and Non-Integrating Image sensors

In an integrating image sensor, the pixel employs a photodetector operating in photovoltaic mode whose signal output is proportional to a charge collected by the pixel during the integration time. This integration process begins with the reset or drainage of all charges present in the pixel, and ends with the readout of the final charge. Given the finite period of time required
between successive readout operations, this type of image sensor cannot be readout arbitrarily, but requires a timing scheme for the reset and the readout operation. Fig. 1.5 illustrates the general architecture of an integrating image sensor.

In non-integrating pixels the output signal is directly proportional to the photocurrent generated in the pixel’s photodetector. This type of pixel does not require a timing scheme, and can be accessed asynchronously like a memory device. An example of a non-integrating pixel is the logarithmic pixel, which features a logarithmic photoelectric transfer curve for a wider optical dynamic range [11]. A major drawback of this type of sensor is that the image quality is typically low, resulting from a high readout noise and a high non-uniformity. Fig. 1.6 illustrates the general architecture of a non-integrating type image sensor with random-access readout.

Fig. 1.5 Architectural view of integrating type image sensor with frame-scan readout [11].
1.5 Fixed Pattern Noise

The non-uniformities of the photodetectors, in-pixel transistors, and the array readout circuitry imprint the output image of an image sensor with a static offset noise, referred to as *fixed pattern noise* (FPN). The effect of fixed pattern noise can be suppressed by subtracting a reference frame from the sensor output. The subtraction of a reference image can be done off-chip utilizing an image-sized memory. Alternatively, there are two readout techniques for integrating image sensors that can effectively remove the effects of FPN: double sampling and correlated double sampling (CDS). FPN is typically expressed as a fraction of the saturation output level of the image sensor.
1.6 Double Sampling Techniques

A readout method called *correlated double sampling* (CDS) stores pixel output immediately after reset operation (pre-integration reset level) and then stores the post-integration signal level to obtain their difference. The difference of these two signals effectively suppresses the fixed pattern noise originating from the signal-path non-uniformities and the $kTC$ capacitor noise associated with the reset operation [12]. This method requires a temporary storage for reset levels of all pixels, which can be implemented with an off-sensor RAM in digital domain or in the pixel itself with a storage capacitor [11]. In *double sampling* readout, the difference between the post-integration signal and the reset level of that pixel after the subsequent reset operation is read out, alleviating the need for the temporary storage; however, double sampling does not suppress the reset ($kTC$) noise as effectively as CDS, since the two samples are obtained after different reset operations [12].

1.7 Dark Current

Impurities, material-boundaries in the substrate, and proton irradiation damage facilitate thermal generation of free carriers, which are indistinguishable from photocarriers [7]. These thermally generated free carriers lead to dark current, which is the spurious photodetector output in absence of illumination. Dark current generation is non-uniform over the focal plane array, contributing to the fixed pattern noise of the array. With integrating image sensors, the amplitude of fixed pattern noise originating from dark current is proportional to the integration time. The dark current induced non-uniformity cannot be corrected by double sampling techniques since the non-uniformity is embedded into the image during the integration time. The only effective suppression method is the subtraction of a reference dark image from the obtained images [11].
1.8 Temporal Noise

Temporal noise sets the fundamental limit on image sensor performance. The main sources of temporal noise in image sensors are photodetector shot noise, $kT$C noise from reset operation, and the amplifier thermal and $1/f$ noise [12]. Among these, only $kT$C noise is treated with the correlated double sampling technique.

1.9 Lateral Diffusion and Lateral Crosstalk

Fig. 1.7 illustrates lateral diffusion of photocarriers in imaging arrays. Lateral diffusion of photocarriers in imaging arrays lead to edge-effect and lateral crosstalk. The edge-effect is the collection of photocarriers along the lateral edge of the photodetector resulting from the lateral diffusion photocarriers, also referred to as peripheral photoresponse or lateral collection. The edge-effect leads to photoactive area that is slightly larger than the physical geometry of the photodetector and is most pronounced in small photodetectors where the perimeter-to-area ratio is relatively large.
Fig. 1.7 An illustration of lateral diffusion of photocarrier in an imaging array.

Whereas the edge-effect describes the sheer increase in the quantity of collected photocarriers as a result of lateral diffusion, *lateral crosstalk* identifies a component within laterally collected photocarriers consisting of stray photocarriers which have diffused out of the pixel site where it was generated. In applications such as solar cells, lateral crosstalk is irrelevant and edge-effect is desirable in order to maximize the quantum efficiency of each pixel. In other applications such as imaging arrays, lateral crosstalk leads to an undesirable blurring effect on the captured image and leads to a fundamental lower boundary on the pixel size below which the effective resolution the image sensor does not improve [13].
1.10 Motivation and Organization of Thesis

Comprehensive models of the CMOS based pixels, including its optical and electrical characteristics, are necessary in order to translate given design objectives to proper choices of technology, pixel architecture, array structure, design layout, etc. The present trend in CMOS technology scaling introduces additional challenges to the CMOS image sensor design [14] and the further underlines the priority of the modeling work.

Presently, compact models of CMOS photodiodes and other types of photodetectors suitable for the circuit-level simulations are not yet available. As a result, overall pixel operations can only be simulated on numerical device simulators, which require detailed information about the underlying fabrication technology. Analogous to development of SPICE models for transistors, development of compact models for CMOS photodetectors can enable designers to direct their attention on design efforts, alleviated from the details of the fabrication process and their relations to the photodetector characteristics. Furthermore, compact models of CMOS photodetectors can also convey relevant information image sensor designers while maintaining the confidentiality of the process details.

Toward this end, improved analyses of CMOS photodiodes and photodiode arrays are presented in this thesis with discussions on the significance of surface recombination, mobility degradation along the Si-SiO₂ interface, epitaxial thickness, and epitaxial-substrate junction. Chapter 2 presents one-dimensional analysis of CMOS photodiode toward the development of a compact model of external quantum efficiency. Chapter 3 presents three-dimensional analysis of edge-effect under the case of uniform illumination intended to provide a first-order estimation of the photocurrent increase resulting from lateral diffusion.
Lateral crosstalk in CMOS imaging arrays deter effective utilization of small pixel sizes (e.g. < 5.0 µm x 5.0 µm) now permitted by technology scaling. An analytical derivation of lateral crosstalk is generally cumbersome and requires substantial approximations to maintain practical expression complexity. Lateral crosstalk in CMOS image sensors is also affected by presence of in-pixel circuitry that further challenges analytical approach. Modular transfer function (MTF) measurements using special test images are commonly used to empirically characterize the effects of lateral crosstalk. Chapter 4 presents an alternative empirical characterization method for lateral crosstalk in CMOS image sensors using focused laser spot illumination. A number of studies have already reported utilizing laser scanning technique to characterize sub-pixel and lateral photoresponse in CMOS imaging arrays. Here, a much simpler setup that is generally sufficient for characterizing lateral crosstalk is illustrated. A discussion of deblurring operations based on the extracted blur model of lateral crosstalk is also presented. A comparative look at the results of the employed linear deconvolution filters illustrates several important trade-offs in devising an appropriate deblurring operation for lateral crosstalk in CMOS image sensors.
Chapter 2

Quantum Efficiency

2.1 Survey of Photodiode Analyses

The classical analyses of photovoltaic devices summarized by van de Wiele [15] illustrate one-dimensional treatments under time-independent excitation for various boundary conditions. More sophisticated analyses that forgo the abrupt junction and quasi-equilibrium approximations were demonstrated by Geist and Baltes [16] who utilized numerical methods to incorporate the effects of graded junctions and the small potential variation across the quasi-neutral regions.

Following one-dimensional analyses of photodiodes and other photovoltaic devices, efforts focused on the effects of lateral diffusion in linear and two-dimensional arrays as they became generalized with CCDs. Numerous studies of lateral diffusion have investigated the edge effect—the collection of photocarriers along the lateral edge of the photodiode—also known as peripheral photoresponse or lateral collection [13], [17]–[26]. Other studies of lateral diffusion have dealt with the lateral crosstalk—the unwanted component within edge-effect consisting of stray photocarriers that have diffused out of the pixel site in which it was generated to be collected by a “wrong” pixel—suggesting a number of ways to minimize this effect [27]–[33]. The effect of lateral crosstalk on the modulation transfer function (MTF) in the context of imaging array has also been investigated [27]–[30]. Several researchers have also proposed a definition called cross-responsivity to characterize lateral crosstalk, which is a discrete point-spread-function (PSF) response of an imaging array resulting from single pixel excitation, analogous to impulse response of a linear shift-invariant system [30]–[33].
Whether characterized with MTF or cross-responsivity PSF, an analytical derivation of lateral
crosstalk is generally cumbersome and requires substantial approximations to maintain practical
expression complexity [17]–[33]. This follows from the observation that lateral photoresponse is
a multidimensional phenomenon consisting of a number of physical processes – photogeneration,
lateral diffusion, bulk and surface recombination, mobility degradation near surface, and 3D
collection fields – that are unwieldy to integrate analytically. Hence, MTF measurements have
been generally adopted to empirically characterize the effects of lateral crosstalk [27]–[29].

This chapter presents an improved one-dimensional analysis of single-junction CMOS
photodiodes (e.g. n⁺-p_epi, nwell-p_epi) including the effects of the high-low doped junction between
the epitaxial layer and the substrate, which had not been treated in the literature except in few
analyses of CCDs [29]. The presented analysis also exploits several characteristics of the modern
CMOS technology that introduce further derivations not covered in previous analyses based on a
general solid-state technology.

2.2 One-Dimensional Analysis

Fig. 2.1 shows the measured spectral response obtained from an n⁺-p_epi photodiode fabricated on
a standard 0.35 μm CMOS process. Also shown in the figure is the theoretical transmittance, (1–
R), of a single amorphous silicon dioxide overlayer between the epitaxial silicon and incident
medium of air, fitted to the interference pattern in the measured spectral response. The theoretical
expression of the reflectance, R(λ), is given by [34]–[36].
\[ R(\lambda) = \frac{n_{\text{SiO}_2}^2 (n_{\text{air}} - n_{\text{SiO}_2})^2 \cos^2 (2\pi p/\lambda) + (n_{\text{Si}} - n_{\text{SiO}_2})^2 \sin^2 (2\pi p/\lambda)}{n_{\text{SiO}_2}^2 (n_{\text{air}} + n_{\text{SiO}_2})^2 \cos^2 (2\pi p/\lambda) + (n_{\text{Si}} + n_{\text{SiO}_2})^2 \sin^2 (2\pi p/\lambda)} \]  

(2.1)

where \( n_{\text{air}}, n_{\text{Si}}, n_{\text{SiO}_2} \) are indices of refraction for air, silicon crystal, and amorphous silicon dioxide, respectively; \( \lambda \) is the wavelength of the incident light; \( p \) is the optical path length difference caused by the silicon dioxide layer given by

\[ p = n_{\text{SiO}_2} \cdot t_{\text{ox}} \cdot \cos \theta \]  

(2.2)

where \( t_{\text{ox}} \) is the thickness of the oxide layer, and \( \theta \) is the angle of transmission with respect to normal assumed to be zero here. (2.1) was fitted to the measured spectral response by varying the thickness of the passivation, \( t_{\text{ox}} \), and a reasonably good match was observed at \( t_{\text{ox}} \approx 6.025 \mu\text{m} \) (Fig. 2.1). Note that the reflectance, \( R(\lambda) \), is minimized when the optical path difference, \( p \), is an odd multiple of \( \lambda/4 \), reducing to

\[ R = \left( \frac{n_{\text{air}} n_{\text{Si}} - n_{\text{SiO}_2}^2}{n_{\text{air}} n_{\text{Si}} + n_{\text{SiO}_2}^2} \right)^2 \]  

(2.3)
The passivation/insulation of a CMOS photodiode may consist of multi-layered stack of oxides, nitrides, and oxynitrides [37]. In image sensor targeted CMOS processes, anti-reflection coatings and colour films may also be employed. For the particular 0.35 µm CMOS technology employed for this investigation, a simple reflection model based on a single layer of glass sufficed in providing a semi-empirical account of the observed reflection pattern. In general cases involving one or more layers of significantly different refractive indices, however, a multi-layer
model of thin-film reflections may likely be necessary to account for the observed reflection pattern in the spectral response [34]–[36].

Considering the values of \((1-R)\) shown in Fig. 2.1, it can be seen that the reflections at the boundaries of the silicon dioxide layer is responsible for a significant loss of incident photons in the absence of any anti-reflection coating. The remaining discrepancy between the two interference patterns may be attributed to a number of causes, including the simplicity passivation/insulation structure assumed, the non-uniformity of the thickness of the oxide caused by presence of contacts and interconnects, the limited precisions of the equipments involved, and even possibly a small misalignment in the test fixture which can lead to significant variations in the observed interference pattern, especially for the short-wavelength illuminations.

Given the substrate bulk is generally more heavily doped than the epitaxial layer, the resulting electric fields inject photocarriers from the substrate into the epitaxial layer. The significance of the substrate contribution of photocarriers in the photoresponse CMOS photodiodes is evident when the maximum possible quantum efficiency of the epitaxial layer alone is compared to the measured spectral response. For an epitaxial layer of thickness, \(t\), the highest quantum efficiency results when all of the absorbed photons lead to photo-collection, yielding

\[
\eta = \int \exp(-z / L_{op}) dz = 1 - \exp(-t / L_{op})
\]

(2.4)

where \(L_{op} = 1/\alpha(\lambda)\). The thickness of the epitaxial layer was estimated to be 3.0 \(\mu\)m with a scanning electron microscope as shown in Fig. 2.2. Fig. 2.3 shows a comparison of the measured spectral response versus (2.4) multiplied by the previously determined values of \((1-R)\). From the comparison it can be observed that the epitaxial layer alone cannot account for all of the
measured photoresponse, especially in the long-wavelength regime of the visible spectrum where the substrate contribution is expected to be the largest. This observation can be summarized by the fact that the characteristic absorption distance of 740 nm light in crystal silicon is approximately 6.0 µm while the epitaxial layer thickness used in present CMOS technology generally ranges from 1.5 to 4.0 µm [37].

Fig. 2.2 Two images of CMOS epitaxial layer taken with a scanning electron microscope in (a) back-scattered mode; (b) secondary-electron mode. In back-scatter mode, the epitaxial-substrate junction is revealed as a distinct bright line.
Fig. 2.3  (a) Measured external quantum efficiency of n⁺-p_epi photodiode – thin line with data points; (b) Theoretical quantum efficiency based on epitaxial collection only – thick line.

Fig. 2.4 Simple one-dimensional description of n⁺-p_epi type photodiode. Note that z-axis represents the dimension along the depth of photodiode. This is to use same notations as the three-dimensional analysis presented in Chapter 3.
Consider Fig. 2.4 depicting the one-dimensional structure of n’-p_{epi} photodiode. The n’-p_{epi} type photodiode was selected as it has a relatively precise junction depth, and hence a more tractable structure. The epitaxial layer is characterized by thickness $t$, uniform doping concentration $N_e$, and minority carrier diffusion length $L_e$. The bulk substrate region is characterized by a different doping concentration $N_s$, and diffusion length $L_s$. The distance between the bottom edge of the photodiode junction and the Si-SiO$_2$ interface is denoted by $d$. The excess minority carrier generation rate from optical excitation, $G(z)$, is given by

$$G(z) = G_o \exp(-\alpha z)$$  \hspace{1cm} (2.5)

The peak photo-generation rate at the surface, $G_o$, can be expressed as

$$G_o = \alpha P_{in} \frac{\lambda}{hc} (1 - R(\lambda))$$  \hspace{1cm} (2.6)

where $P_{in}$ is the incident power density, $h$ is Plank’s constant, and $c$ is the speed of light.

The proposed boundary conditions for the excess carrier concentration in the epitaxial layer are

$$\hat{n}_e(z = d) = 0$$  \hspace{1cm} (2.7)

---

$^\dagger$ The subsequent analysis and discussions, however, also applies to other single junction photodiodes such as nwell-p_{epi}. Section 2.3 presents a set of semi-empirical characterization procedures including the empirical extraction of the effective photodiode junction depth, which may accommodate the effects of graded photodiode junctions.
where \( \Phi_s \) is the flux of diffused excess minority carriers from the substrate injected into the epitaxial layer by the high-low junction.

Here, we solve the steady-state diffusion equation (i.e. Continuity equation with zero electric field approximation in quasi-neutral region) with the boundary conditions (2.7) and (2.8) and simplify the expression to obtain

\[
\dot{n}_e(z-d) = \frac{G_{L_{op}}}{D_e} \left( L_{op} \left( \exp(-d/L_{op}) - \exp(-z/L_{op}) \right) - L_e \exp(-t/L_{op}) \sinh \left( \frac{z-d}{L_e} \right) \right) + \Phi_s \frac{L_e}{D_e} \sinh \left( \frac{z-d}{L_e} \right)
\]

for \( z \geq d \). (2.9)

Deriving the photocurrent expression from (2.9),

\[
J_{ph} = qD_e \frac{\partial \dot{n}_e}{\partial z} \bigg|_{z=d} = qG_{L_{op}} \left( \exp(-d/L_{op}) - \exp(-t/L_{op}) \right) + q \Phi_s
\]

(2.10)

The resulting photocurrent expression in (2.10) is simply the integration of all optical excitation in the region of analysis (i.e. \( z=d \) to \( z=t \)) plus the photocurrent from the substrate. This follows from the fact that the minority carrier diffusion lengths of epitaxial silicon in modern CMOS
processes are typically in the order of hundreds of micrometers [37] as compared to several micrometers that typify epitaxial layer thickness.

The properties of a high-low junction are mainly characterized by the doping ratio across the junction, which largely determines the equilibrium statistics [38]. Under the equilibrium condition, the electric fields resulting from doping gradient lead to injection of minority carriers from the higher doped side to the lower, and the build up of injected carriers in the lower doped side lead to back diffusion carriers to the higher doped side, analogous to PN junctions.

For the case of CMOS photodiodes (or photogate), the high carrier mobility of the epitaxial layer permits photodiode junction to sink the injected photocarriers faster than the rate of photocarrier arrival from the bulk substrate, characterized by lower carrier mobility and lifetime. Fig. 2.5 shows the steady-state excess minority carrier distribution along the vertical dimension of $n^+\text{-}p_{\text{epi}}$ photodiode illuminated by 740 nm light as simulated by numerical device simulator Medici based on parameters reported for the employed 0.35 µm technology.† The 740 nm illumination ($L_{\text{op}} \approx 6.0 \, \mu\text{m}$) was selected to illustrate the case where the light absorption trails deep into the substrate leading to significant substrate contribution of photocarriers. It can be seen that despite the high rate of optical generation and the additional photocarrier contribution from the substrate, the carrier density in the epitaxial layer is relatively low compared to the substrate region. This follows from the observation that the carrier flux across the epitaxial-substrate junction is limited by the rate of photocarrier arrival from the bulk substrate to the high-low junction, as the diffusion process is significantly slower in the substrate due to the lower carrier mobility. This property of epitaxial-substrate junction inhibits the back diffusion of

† Medici is a product of Avant! Corporation.
photocarriers to the substrate side, which causes the high-low junction to effectively behave as an efficient carrier sink to the substrate photocarriers. Consequently, the flux of photocarriers from the substrate side, $\Phi_s$, can be suitably approximated with the boundary condition,

$$\dot{n}_s(t) = 0$$  \hspace{1cm} (2.11)

Also, considering the typical thickness of the substrate bulk,

$$\dot{n}_s(z) \rightarrow 0 \text{ as } z \rightarrow \infty$$  \hspace{1cm} (2.12)

Solving the diffusion equation for the substrate region, we obtain

$$\dot{n}_s(z) = \frac{G_s \exp(-t / L_{op})}{D_s} \left( \frac{L_s L_{op}^2}{L_s^2 - L_{op}^2} \right) \left( \exp \left( \frac{t - z}{L_s} \right) - \exp \left( \frac{t - z}{L_{op}} \right) \right) \text{ for } z \geq t$$  \hspace{1cm} (2.13)

where

$$\Phi_s = D_s \frac{\partial \dot{n}_s}{\partial x} \bigg|_{z=t} = G_s \exp(-t / L_{op}) L_{op} L_s \left( \frac{L_s - L_{op}}{L_s^2 - L_{op}^2} \right)$$  \hspace{1cm} (2.14)

Finally, substituting (2.14) into (2.10), the resulting photocurrent expression is given by

$$J_{ph} = q G_s L_{op} \left( \exp(-d / L_{op}) - L_{op} \exp(-t / L_{op}) \left( \frac{L_s - L_{op}}{L_s^2 - L_{op}^2} \right) \right)$$  \hspace{1cm} (2.15)
Fig. 2.5  Excess minority carrier density in the substrate as a function of the substrate to epitaxial doping ratio (\( N_s/N_e \)) obtained by numerical device simulation using Medici. The plots are normalized to the peak excess minority carrier generate rate, \( G(z=0)=G_o \). The simulation parameters were based on the employed 0.35 µm CMOS technology.

A set of structural and semiconductor parameters based on the reported SPICE parameters of the employed 0.35 µm CMOS technology were identically applied to the derived photocurrent expression in (2.15) and to numerical simulations using Medici.† The Medici simulation file is provided as Appendix B. Fig. 2.6 shows the result of the comparison. The photocurrent expression demonstrated an excellent agreement with the numerical device simulation with a minor discrepancy (0.53% RMSE).
Fig. 2.6 Comparison of the derived expression of photocurrent (gray line) and numerical simulation using Medici at $N_s/N_e=10$ (cross dots) expressed as internal quantum efficiency. Both numerical simulation and analytical calculations were based on an identical set of material and structural parameters. The comparison yielded discrepancy of only 0.53% RMSE. Some parameters such as the minority carrier lifetime of the bulk substrate were estimated based on reported measurements and typical parameter values. The slight irregularities in otherwise smooth spectral response arise from those of the absorption coefficients for silicon [36], which were also identically used in the comparison.

2.3 Suggested Empirical Parameters

While several semiconductor parameters such as the minority carrier lifetime of the bulk substrate were estimated in carrying out the comparison between (2.15) and numerical simulations, their precise values were needed in order to compare the analytical expression to the

† The photocurrent was taken at $z = d$ in the numerical simulation in accordance with the analysis.
measured spectral response. Given that the minority carrier diffusion coefficient and the minority carrier lifetime of the substrate are not generally reported, the substrate diffusion length in (2.15) is replaced with an empirical fitting parameter, $L_{s,fitted}$, as a practical alternative to extracting the semiconductor properties of the substrate. It was found that fitting $L_{s,fitted}$ also effectively compensates for any minor error in the estimation of the epitaxial layer thickness. Fig. 2.7 shows the result of fitting the substrate diffusion length to the measured spectral response. The substrate diffusion length of $L_{s,fitted} = 4.0 \, \mu m$ was extracted from the best fit. This is a reasonable value since the falloff of the measured quantum efficiency from the fitted $(1-R)$ curve in the long-wavelength regime (Fig. 2.1) indicates that the actual substrate diffusion length is not significantly longer than $(L_{opt}-t)$ which is approximately $6.0 - 3.0 = 3.0 \, \mu m$ at 740 nm light.

At the short-wavelength end of the spectrum (Fig. 2.7) the falloff of the theoretical expression from the measured spectral response is due to the unaccounted collection within the photodiode depletion region and the diffused photocarriers from the “n+” quasi-neutral region. Assuming that recombination in the photodiode depletion region is negligible, the collection in this region can be incorporated to the analysis by integrating the optical generation and adding the integral to the total collection. This is accomplished by redefining $d$ in (2.15) as the depth of the upper edge of the depletion region, which can be approximated using the metallurgical junction depth typically reported as a SPICE parameter (i.e. XJ). However, the employed value of metallurgical junction depth needs to be very precise in order to accurately simulate the short-wavelength response. For instance, the characteristic optical absorption distance for 400 nm light is only
about 80 nm, and given the typical junction depth of 100 nm, an inaccurate determination of the junction depth by 10 nm can cause the short-wavelength response to deviate as much as about 10%, owing to the proximity of the n⁻-p_epi junction to the incident surface. Based on these considerations, an empirical extraction of the effective junction depth, \(d_{\text{fitted}}\), is suggested as it would automatically account for the collection in the depletion region and compensate for any error in the junction depth. Hence, replacing \(d\) in (2.15) with \(d_{\text{fitted}}\), we obtain
\[
J_{ph} = qG_o L_{op} \left\{ \exp\left(-d_{\text{fitted}} / L_{op}\right) - L_{op} \exp(-t / L_{op}) \left( \frac{L_{s,\text{fitted}} - L_{op}}{L_{s,\text{fitted}} - L_{op}} \right) \right\}
\]  

(2.16)

Fig. 2.8 shows the result of the empirical fitting at \( L_{s,\text{fitted}} = 4.0 \mu m \) and \( d_{\text{fitted}} = 50 \) nm. The empirical extraction of the effective junction depth leads to a close fit at the short-wavelength regime. An additional analysis for the collection in the heavily doped “n+” quasi-neutral region is not included as the empirical extraction of the effective junction seemed to have provided a sufficient fit at the short-wavelength regime. Hence, for simplicity, the effective junction depth, \( d_{\text{fitted}} \), is used to empirically account for the collection in the “n+” quasi-neutral region as well. The reported junction depth (XJ) for the employed technology is 100 nm, twice the fitted to value of \( d_{\text{fitted}} = 50 \) nm, indicating that the “n+” quasi-neutral region provides a significant contribution of photocarriers in the short-wavelength regime of the visible spectrum. It is also useful to note that the empirical extraction of the effective junction depth permits the presented semi-empirical model to accommodate other single junction photodiode types such as nwell-pepi type photodiodes. The photodiode junction profile is unimportant in the prescribed semi-empirical characterization of short-wavelength response since only the location of the quantum efficiency falloff in the short-wavelength region is of the interest. The obvious condition, however, is that the shape of falloff generally follows an exponential decay, i.e. \( \exp(-d_{\text{fitted}}/L_{op}) \).

An improved model of optical reflections in CMOS overlayers appears necessary in order to further minimize the remaining discrepancies.
2.4 Summary

A simple model of optical reflection based on a single layer of amorphous silicon dioxide provided a reasonable fit to the observed interference pattern in spectral response of n^+-p_{epi} photodiode fabricated on a standard 0.35 µm CMOS technology. The presented one-dimensional analysis of CMOS photodiode including the effect of the epitaxial-substrate junction showed a good agreement with numerical device simulations (Fig. 2.6). Two empirical parameters were
suggested as a practical alternative to the parameter extractions that need to be highly precise. The final semi-empirical model of quantum efficiency demonstrated a close fit with the measured spectral response (Fig. 2.8). An improved model of optical reflections in CMOS overlayers appears necessary in order to further minimize the remaining discrepancies.
Chapter 3

Lateral Photoresponse

In Chapter 2, an improved one-dimensional analysis of CMOS photodiode was derived in which the effect of the high-low junction of the epitaxial layer and the substrate is considered. The analytical solution was verified with numerical simulations based on parameters extracted from a standard 0.35 µm CMOS process and two empirical parameters are suggested to offset the unavoidable inaccuracies in the extracted parameter values.

The organization of this chapter is as follows. In section 3.1, the investigation of lateral photoresponse using linear photodiode arrays and numerical device simulations is presented, illustrating the importance of surface recombination and mobility degradation along Si-SiO₂ interface. Section 3.2 presents a novel three-dimensional analysis of lateral photoresponse based on the earlier work of Levy [31]. Section 3.3 illustrates a more reliable approach of empirically modeling lateral photoresponse based on test structures employing linear photodiode arrays. This is followed by a method of utilizing empirically characterized two-dimensional edge-effect to draw useful limits for the case two-dimensional rectangular photodiode arrays.

3.1 Lateral Photoresponse

The lateral diffusion of photocarriers generally leads to a photoactive area that is larger than the geometric area of the photodiode junction. This phenomenon is known as the edge effect [17]–[26] and is most pronounced in small photodiodes where the perimeter-to-area ratio is relatively large. While the edge-effect describes the sheer increase in the quantity of collected photocarriers
as a result of lateral diffusion, lateral crosstalk is a component within lateral collection consisting of stray photocarriers that have diffused out of the pixel site in which it was generated to be collected by a “wrong” pixel. In imaging arrays, lateral crosstalk can significantly degrade the contrast of the sampled image, and presents a lower boundary on the size of the pixel below which the resolution of the sampled image does not significantly improve with the sampling frequency of the array [13].

This chapter presents an analysis of edge-effect under uniform illumination. The intention of this analysis is to provide a model of photocurrent as a function of the photodiode separation distance for linear and two-dimensional arrays. An investigation of lateral crosstalk in CMOS image sensors is presented in Chapter 4.

In order to quantitatively investigate the edge-effect in CMOS n^-p_epi photodiodes, a set of linear photodiode arrays was fabricated as shown in Fig. 3.1. The fabricated arrays consist of identical 30 µm x 5 µm strips of n^-p_epi photodiodes with a specific separation distance, s, assigned to each array, which ranged from 0.6 µm to 12 µm. The photodiodes were connected in parallel to increase its signal strength and to average out the mismatches. Fig. 3.2 shows the photocurrent measured from each array under uniform illumination as a function of s. The plotted photocurrents are normalized to the one-dimensional (or “areal”) photocurrent density measured from a single large photodiode under identical set of illuminations multiplied by the photodiode junction area of 150 µm². It can be seen from Fig. 3.2 that the three plots corresponding to the wavelengths 440 nm, 540 nm, and 660 nm are essentially identical, indicating that the proportional increase in the photocurrent from a given increase in separation distance is essentially identical for the different wavelengths. This observation can be appreciated from the fact the vertical profile of an optical generation does not alter the percentage of photocarriers
diffusing laterally except very slightly through bulk recombination.† This is also true for the case of 660 nm wavelength illumination for which a significant portion of the generated photocarriers recombine in the substrate bulk; however, only the photocarriers that drift into the epitaxial layer contribute to the observed photocurrent, and the injected carriers will diffuse in a similar manner to those generated in the epitaxial layer.

Fig. 3.1 (a) A micrograph of the fabricated linear photodiode array, and (b) an illustration of the layout. Each array consisted of nearly one hundred 30 µm x 5 µm n⁺-p_epi photodiode strips connected in parallel. A guard structure was used to prevent collection of stray photocarriers generated elsewhere in the test chip.

† While the excess minority carrier diffusion length can slightly decrease with higher illumination intensity through bulk recombination, it will likely remain much longer than the typical photodiode separation distances, leading to only a very small fraction of recombined photocarriers in lateral photoresponse.
Fig. 3.2  Measured photocurrent from the linear photodiode arrays (Fig. 3.1) plotted as a function of the photodiode separation distance illustrating the edge-effect. The photocurrents were normalized to photocurrent density obtained from a single large photodiode. The applied illumination intensity ranged from 41 µW/cm² to 73 µW/cm² for different wavelengths. A single exponential decay was fitted to characterize the general trend. The linear plot is a simple one-dimensional approximation of the edge-effect using the photocurrent density.

Fig. 3.2 also shows a single exponential decay function, \( f(s) = 1 - e^{-s/a} \), fitted to the measured plots. The characteristic length of the fitted exponential function was found to be approximately 5.5 µm, which is considerably shorter than the minority carrier diffusion length of the epitaxial layer, \( L_{ce} \), indicating that a significant portion of the lateral diffusion occurs along the Si-SiO₂ interface where the diffusion is hindered by surface recombination and the mobility degradation. Fig. 3.3 compares the measured lateral photoresponse with numerical simulations based on
varying interface quality. It can be seen that both surface recombination and the mobility degradation can significantly affect the lateral collection and account for the deviation of the measured lateral photoresponse from the hypothetical case of a defect-free Si-SiO₂ interface.

Fig. 3.3 Measured lateral photoresponse (data points) versus numerical simulation. Various surface recombination velocity and mobility degradation near the Si-SiO₂ were simulated to illustrate their significance. Since the actual surface recombination velocity of the employed 0.35 μm CMOS technology is not reported, the dotted lines are simulation results using surface recombination velocity of 10 cm/s while the solid lines correspond to 10⁵ cm/s. The mobility near the Si-SiO₂ interface was taken at 740, 700, 680, and 650 cm²/Vs, corresponding to the four contours shown above (with 650 cm²/Vs providing the closest fit). The straight plot corresponds to the hypothetical case of defect free Si-SiO₂ interface.
Appendix C provides photocurrent measurements from annular (or donut-shaped) photodiodes which also demonstrate the significance of surface recombination in CMOS photoresponse.

A simple one-dimensional approximation of the edge-effect based on the areal current density (from single large photodiode) multiplied by a hypothetical photoactive area encompassing all area between the photodiodes is plotted in Fig. 3.2. It can be seen that this simple one-dimensional approximation can be used to estimate the lateral photoresponse when the diodes are spaced relatively close together. The measured lateral photoresponse is slightly higher than the one-dimensional approximation at small diode separation distances (Fig. 3.2) likely due to the effect of the side-wall junctions.

While the comparison of the measured data and numerical device simulations suggests that the edge-effect in CMOS photodiodes is significantly affected by surface recombination and mobility degradation along the Si-SiO$_2$ interface, the reported treatments of multidimensional diffusion equation generally approximate the surface conditions with infinite surface recombination velocity [17], [24], [25], [29]–[33]. Others that accommodate finite surface recombination velocity do so by employing numerical methods, yet generally do not account for the mobility degradation along the Si-SiO$_2$ interface [21], [23]–[25]. In a multidimensional analysis, it is cumbersome to directly incorporate these effects into an algebraic treatment of the diffusion equation, and the resulting expressions are generally unmanageable without assuming special cases that lend to simplified boundary conditions; in addition, beyond deriving an accurate analytical model of the edge-effect, the required physical parameters including surface recombination velocity and carrier mobility under the CMOS field oxide are not typically reported, bringing forth additional parameter extraction needs which may not be readily addressable. Despite these limitations and issues in analytical treatments of photodiode arrays, a
novel three-dimensional analysis of edge-effect based on earlier work of Levy [31] is introduced in the next section because it was found to provide a reasonably good analytical solution. Since the quality of Si-SiO₂ interface for both gate and field oxides can vary significantly from process to process and sway the effectiveness of the analytical approximation, a more reliable approach is to empirically characterize edge-effect (as well as other lateral photoresponse phenomena such as lateral crosstalk) for each given technology. An empirical characterization method employing a set of linear photodiode arrays is therefore also illustrated in section 3.3.

3.2 Three-Dimensional Analysis of Lateral Photoresponse

The notational framework for two-dimensional rectangular photodiode array used by Levy [31] is adopted in our analysis (Fig. 3.4). An analytical solution of the edge-effect is derived by solving the excess carrier distribution under selective illumination of the photodiode array. The diffusion equation under steady-state for a p-type epitaxial layer is given by

\[ \nabla^2 \hat{n} + \frac{\hat{n}}{L_c^2} \frac{G(x,y,z)}{D_c} = 0 \]  

(3.1)

where \( D_c \) and \( L_c \) denote the bulk minority carrier diffusion coefficient and diffusion length of the epitaxial layer, respectively.

Under uniform illumination, the edge-effect experienced by each photodiode is identical, and the region of analysis may be reduced to a single pixel site as defined by \((-L_X-R_X) < x < (L_X+R_X)\) and \((-L_Y-R_Y) < y < (L_Y+R_Y)\). In order to analytically isolate the lateral component within the
substrate diffusion current that leads to edge-effect, the area of illumination is selected as follows:

\[
H(x, y) = \begin{cases} 
0 & \text{for } (2mL_X - R_X < x < 2mL_X + R_x; \\
2nL_Y - R_Y < y < 2nL_Y + R_Y) \\
1 & \text{elsewhere}
\end{cases}
\]  

(3.2)

where \( m \) and \( n \) are integers. This mask function represents the case where all photodiode junction areas are blocked from illumination (Fig. 3.4). The resulting optical generation profile is given by

Fig. 3.4 The top and cross sectional views of two-dimensional rectangular photodiode array. The theoretical optical masks overlay all photodiode areas in order to expose only the epitaxial surface areas in between. This leads to photocarrier generation responsible for edge-effect under uniform illumination.
The source term in (3.3) identifies the photocarriers responsible for the edge-effect. Note that the contribution from the excitation within photodiode area is considered the “areal” collection, which is distinguished from the lateral collection responsible for the edge-effect. While photocarriers generated under photodiode junctions also contribute to lateral diffusion, their diffusion cancels out under uniform illumination and do not influence the observed edge-effect. It may be worthwhile to note that investigations of lateral crosstalk, on the other hand, cannot neglect the lateral diffusion of photocarriers generated underneath the photodiode junction as done here [27]–[33].

In order to obtain a full analytical solution of the edge-effect (Fig. 3.2), the analysis is limited to the case where the adjacent photodiodes are closely spaced as compared with their length and width such that a smooth boundary condition at \( z = d \) plane may be assumed [31], such that

\[
\hat{n}(z = d) = 0
\]

(3.4)

Also, the symmetry of the array structure and of the illumination implies that

---

† The implied assumption is that photoresponse of CMOS photodiodes can be, in practice, modeled as a linear process. Despite non-linearity imposed by several physical phenomena related to carrier transportation and recombination that render the process of photoresponse to be strictly a non-linear system, the general observation that the process of photoresponse can be essentially modeled as a linear system within practical limits of operating parameters is adopted. Note that concepts such as quantum efficiency and MTF are also used with the implied assumption of linear photoresponse.

‡ At large photodiode separation distances, the approximation afforded by (3.4) is no longer valid; however, it will be seen that the neglected mobility degradation along the \( z = 0 \) plane somewhat offsets the underestimation resulting from (3.4).
\[
\frac{\partial \hat{n}}{\partial x}_{z=0} = \frac{\partial \hat{n}}{\partial y}_{y=0} = 0
\]  

(3.5)

\[
\frac{\partial \hat{n}}{\partial x}_{z=L_z} = \frac{\partial \hat{n}}{\partial y}_{y=L_y} = 0
\]  

Given the wavelength independence of the edge-effect, the analysis may be limited to short-wavelength illumination whose characteristic absorption depth is much shorter than the epitaxial thickness, that is,

\[L_{op} \ll t\]  

(3.6)

Another boundary condition for the backside of the epitaxial layer is considered. The carrier reflection typically used with potential wells can be used to approximate the boundary condition at the epitaxial-substrate junction, that is,

\[\frac{\partial \hat{n}}{\partial z}_{z=0} = 0\]  

(3.7)

This boundary condition provides us with a useful case where the characteristic absorption depth of the illumination wavelength is comparable to the epitaxial layer thickness, and the substrate contribution is relatively small in comparison to the total photocurrent.

From the boundary conditions (3.4), (3.5), and (3.6), we have derived the following excess carrier distribution:
\[ n(x, y, z - d) = \frac{G_o}{D_n} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left( \frac{A_{mn}}{L_{ep}^2 - Q_{mn}^2} \cos(m\pi x/L_x) \cos(n\pi y/L_y) \left(e^{-z/L_{ep}} - e^{-Q_{mn}z}\right) \right) \text{ for } z \geq d \quad (3.8) \]

where \( Q_{mn} \) is defined by

\[ Q_{mn} = \left[ \frac{m^2\pi^2}{L_x^2} + \frac{n^2\pi^2}{L_y^2} + \frac{1}{L_{ep}^2} \right]^{1/2} \quad (3.9) \]

and \( A_{mn} \) is defined by

\[ A_{mn} = \begin{cases} \frac{L_x L_y - R_x R_y}{L_x L_y} & m = n = 0 \\ \frac{-2R_x \sin\left(\frac{m\pi R_x}{L_x}\right)}{m\pi L_x} & m \neq 0, n = 0 \\ \frac{-2R_y \sin\left(\frac{n\pi R_y}{L_y}\right)}{n\pi L_y} & m = 0, n \neq 0 \\ \frac{-4 \sin\left(\frac{m\pi R_x}{L_x}\right) \sin\left(\frac{n\pi R_y}{L_y}\right)}{mn\pi} & m \neq 0, n \neq 0 \end{cases} \quad (3.10) \]

The photocurrent derived from the excess carrier distribution at \( z = d \) in (3.8) is given by

\[ J_{ph, lateral} = qG_o e^{-d/L_{ep}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{A_{mn}}{L_{ep}^2 - Q_{mn}^2} B_{mn} (Q_{mn} - L_{ep}^2) \quad (3.11) \]

where
Note that the photocurrent can be obtained by performing the series summation over a large number of terms (about 50 for better than 1 percent accuracy) without resorting to tedious numerical calculations. The excess carrier distribution derived from the boundary conditions (3.4), (3.5), and (3.7) is given by

\[
\hat{n}(x, y, z-d) = \frac{G_e}{D_n} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left( \frac{A_{mn}}{L_{op}^2 - Q_{mn}^2} \right) \left( \frac{\sinh(m\pi x/L_x) \cos(n\pi y/L_y)}{2C_{mn} \sinh(Q_{mn} z) + e^{Q_{mn} z} - e^{-Q_{mn} z}} \right)
\]

for \( z > d \), where

\[
C_{mn} = -\frac{1}{2} \cosh(Q_{mn} l) \left( e^{Q_{mn} l} + e^{-Q_{mn} l} \right)
\]

The diffusion driven photocurrent resulting from the excess carrier distribution in (3.13) is given by

\[
J_{ph, lateral} = qG_e e^{-d/L_{op}} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left( \frac{A_{mn} B_{mn}}{L_{op}^2 - Q_{mn}^2} \right) \left( 2C_{mn} Q_{mn} + Q_{mn} + L_{op}^{-1} \right)
\]
Fig. 3.5 compares the measured lateral photoresponse with the derived photocurrent expression in (3.11) and (3.15) evaluated at \( \lambda = 540 \text{ nm (} L_{op} = 0.91 \mu\text{m}) \) and \( \lambda = 645 \text{ nm (} L_{op} = t = 3.0 \mu\text{m}) \), respectively, in accordance with the assumed backside boundary conditions. It can be seen that the infinite surface recombination velocity approximation leads to a slight underestimation at small diode separation distances while the neglected mobility degradation near the Si-SiO\(_2\) interface eventually leads to a minor overestimation at large diode separation distances. Note that, while (3.11) and (3.15) compare reasonably well with the measured data, this may not hold true for other CMOS technologies with different Si-SiO\(_2\) interface characteristics. In addition, this analysis favors photodiode types with shallow-junctions (e.g. n\(^+\)-p\(_{eps}\)), as it does not account for lateral diffusion in the region \( 0 < z < d \). Hence, a more reliable approach is to empirically characterize the edge-effect (and other lateral photoresponse phenomena such as lateral crosstalk) using a set of test photodiode arrays, and iterating the process for each photodiode-type of interest.
Fig. 3.5 Comparison of the measured edge-effect and the derived lateral photocurrent expressions in (3.11) and (3.15) evaluated at $\lambda=540\text{nm}$ ($L_{op}=0.91\ \mu\text{m}$) and $\lambda=645\text{nm}$ ($L_{op}=t=3.0\ \mu\text{m}$), respectively.

### 3.3 Empirical Characterization of Edge-Effect

The two-dimensional edge-effect shown in Fig. 3.2 can be characterized empirically by using a set of linear arrays consisting of long strips of photodiodes as illustrated in Fig. 3.1. It was found that the normalization of the measured photocurrent to the corresponding areal photocurrent density allows the edge-effect to be approximated independent of the illumination wavelength.
and intensity. In order to express the two-dimensional edge-effect independent of the dimensions of the test array, the following definition is proposed:

\[
E_{2D}(s) = \frac{J_{EE,2D}(s)}{J_{ph,area}} \text{ [\mu m]}
\]  

(3.16)

where

\[
J_{EE,2D}(s) = \frac{I_{ph}(s) - J_{ph,area} \times A}{P} \left[ \frac{A}{\mu m} \right],
\]

(3.17)

\(I_{ph}(s)\) is the measured photocurrent per single photodiode strip of the linear array (Fig. 3.1), \(s\) is the photodiode separation distance between adjacent photodiodes, \(J_{ph,area}\) is the photocurrent density (measured from a single large photodiode), \(A\) is junction area of the photodiode strip (i.e. 150 \(\mu m^2\) in Fig. 3.1), and \(P\) is the photodiode perimeter length (i.e. 60 \(\mu m\) in Fig. 3.1). Fig. 3.6 plots the measured edge-effect expressed as (3.16). A single exponential function, \(f(s) = 1 - e^{-s/\alpha}\), is fitted to the measured results although a more elaborate characterization technique may be used to obtain a better fit. This single empirical curve characterizes the two-dimensional edge-effect, and the process can be repeated for each photodiode-type of interest.

---

† The experiment should employ wavelengths across visible spectrum to obtain a reliable convergence of the measurements. A careful experiment design should also reveal the limits beyond which this general trend is no longer valid.
The measured edge-effect can be compared with the derived lateral photocurrent expressions to ensure that the measurements are reasonably close to these analytical approximations. Substituting the lateral photocurrent expression in (3.11) into (3.16),

\[
E_{2D}(s) = \frac{J_{ph,lateral}}{G_{op}} \frac{I}{8R_x R_y (R_x + R_y)}
\]  
(3.18)

Fig. 3.6 Measured edge-effect characterized as \(E_{2D}\) defined by (3.16) and the two derived analytical approximations (3.18) and (3.20) based on the derived lateral photocurrent expressions in (3.11) and (3.15), respectively.
where the initial assumption in (3.6) allows $J_{\text{ph,area}}$ to be approximated by $G_0L_{\text{op}}$. With the photocurrent expression in (3.15),

$$E_{2D}(s) = \frac{J_{\text{ph,lateral}}}{J_{\text{ph,area}}} \frac{I}{8R_x R_y (R_x + R_y)}$$

(3.19)

Here, $J_{\text{ph,area}}$ cannot be approximated by $G_0L_{\text{op}}$ given that the analysis assumes a wavelength whose characteristic absorption depth is comparable to the epitaxial layer thickness. Hence, $J_{\text{ph,area}}$ from one-dimensional characterization may be employed, or in the absence of experimental data $J_{\text{ph,area}}$ may be approximated by $G_0L_{\text{op}}(1-e^{-s/L_{\text{op}}})$, yielding,

$$E_{2D}(s) = \frac{J_{\text{ph,lateral}}}{G_0L_{\text{op}}(1-e^{-s/L_{\text{op}}})} \frac{I}{8R_x R_y (R_x + R_y)}$$

(3.20)

Note that if the photodiode strips are separated along the x-axis, then the diode separation distance is given by $s=(L_X-R_X)$, and $E_{2D}(s)$ may computed for various value of $s$ by substituting values for $L_X (> R_X)$. Fig. 3.6 also plots the calculated values from (3.18) and (3.20) evaluated at $\lambda = 540$ nm and $\lambda = 645$ nm, respectively, in accordance with the assumed backside boundary conditions. While these analytical solutions show a reasonable agreement with the empirical data, it is important to note that their accuracy is limited by the assumptions of closely spaced photodiodes and shallow photodiode junction.

As previously noted, two-dimensional edge-effect characterized by the linear photodiode arrays can also be applied to the case of two-dimensional photodiode arrays. Consider the
diagram in Fig. 3.4. The photocurrent contribution from region denoted as “A” approaches a maximum limit of $2R_Y J_{EE,2D}(L_X-R_X)$ as $R_Y \to \infty$. The photocurrent contribution from region “B” also approaches a maximum limit of $2R_X J_{EE,2D}(L_Y-R_Y)$ as $R_X \to \infty$. The contribution from region “C”, on the other hand, may be overestimated by $kJ_{EE,2D}(l)$ where $k = \min(L_X-R_X, L_Y-R_Y)$ and $l = \max(L_X - R_X, L_Y - R_Y)$. Therefore, the lateral photocurrent experienced by each rectangular photodiode, $I_{EE,3D, \text{rect}}$, is less than

$$I_{EE,3D, \text{rect}} < 4 \left( R_Y J_{EE,2D} (L_X - R_X) + R_X J_{EE,2D} (L_Y - R_Y) + kJ_{EE,2D} (l) \right)$$  \hspace{1cm} (3.21)$$

It can be seen that the lateral photocurrent approaches the theoretical maximum in (3.21) as $R_X, R_Y \to \infty$ and $(L_X-R_X), (L_Y-R_Y) \to 0$, or when photodiodes are closely spaced together in relation to their XY dimensions. Fig. 3.7 compares the maximum limit in (3.21) with numerical simulation results obtained by Davinci.$^\dagger$ The simulation is based on the fitted parameters of the employed 0.35 $\mu$m technology. The simulation file is provided as Appendix D. Fig. 3.7 suggests that the linear extension of the two-dimensional characterization of edge-effect in (3.21) provides a reasonable approximation of the three-dimensional edge-effect only in the limited case where the photodiodes are closely spaced in relation to its dimensions.

$^\dagger$ Davinci is a product of Avant! Corporation.
Fig. 3.7  The device simulation of two-dimensional array of 5 $\mu$m x 5 $\mu$m photodiodes at various separation distances (shown as dots) compared to the maximum limit expression of (3.21) (shown as line). The photocurrents are normalized to photocurrent density multiplied by 5 $\mu$m x 5 $\mu$m area. $J_{EE,2D}(s)$ is taken from the empirical characterization of the two-dimensional photodiode array shown in Fig. 3.2.

3.4 Summary

The investigation of lateral photoresponse using linear photodiode arrays (Fig. 3.1) revealed the significance of finite surface recombination velocity and mobility degradation along the Si-SiO$_2$ interface (Fig. 3.3). A three-dimensional analysis of edge-effect was presented under the
assumptions of closely spaced photodiodes and shallow junction depth. These assumptions permitted the analysis to neglect the surface recombination and diffusion of photocarriers along Si-SiO₂ interface. Despite these approximations a reasonable agreement with the measured data was observed even at relatively large photodiode separation distances (Fig. 3.5). Yet, given the significance of surface recombination and mobility degradation along the Si-SiO₂ interface, it was concluded that the derived analytical models of edge-effect (as well as many other similar solutions that do not account for these phenomena) are only useful to the extent of providing an approximate analytical solution whose accuracy will vary from employed technology to another. An empirical characterization of edge-effect based on actual measurements is hence recommended as a more reliable alternative.

Two-dimensional characterization of edge-effect using a set of linear photodiode arrays (Fig. 3.1) was proposed as a practical alternative to implementing test structures consisting of two-dimensional arrays. A linear extension of the two-dimensional characterization of edge-effect to the three-dimensional case demonstrated in (3.21) provided a reasonable approximation only in the limited case where the photodiodes are closely spaced in relation to its dimensions.
In this chapter an alternative method for empirical characterization of lateral crosstalk is presented with a demonstration of cross-responsivity PSF measurement using a laser spot illumination. A number of studies have already reported utilizing laser scanning technique to characterize sub-pixel and lateral photoresponse in CMOS imaging arrays [39]–[42]. Here, a much simpler setup that is generally sufficient for measuring cross-responsivity PSF is illustrated. A demonstration of deblurring operations based on the obtained blur model of lateral crosstalk is also provided. A comparison of several linear deconvolution filters is used to illustrate the tradeoffs in sharpness restoration, high-frequency noise amplification and the intensity clipping effect in the design of a linear deblurring operation for lateral crosstalk.

Section 4.1 provides the measurement setup and a discussion of the measured cross-responsivity PSFs obtained from four CMOS image sensors. Section 4.2 demonstrates the deblurring operation with several well-known linear deconvolution filters. The results from the deconvolution filters are examined and compared, and some considerations in designing a linear solution for lateral crosstalk are illustrated. Finally, section 4.3 deals with additional topics for further discussion.

4.1 Experiment Results

Consider a linear model of a CMOS image sensor depicted in Fig. 4.1. $H_{\text{offchip-optics}}$ represents the effects of off-chip optics such as one or more lenses while $H_{\text{onchip-optics}}$ represents the effects of
on-chip optics, which may include anti-reflection coating(s), micro-lenses, and dielectric layers [6], [11], [43], [44]. $H_{\text{diffusion}}$ represents the effects of photogeneration and diffusion of photocarriers [18]–[29]. $H_{\text{integration}}$ represents the spatial (and temporal) integration of photocarriers along the area of the photosensitive device at each pixel [39]. Assuming a square pixel with pitch, $p$, the spatial Nyquist frequency in each dimension is $f_{\text{Nyquist}} = 1/2p$ and scenes with spatial frequencies greater than $f_{\text{Nyquist}}$ cannot be reliably reproduced. $H_{\text{offchip-optics}}$, $H_{\text{onchip-optics}}$, $H_{\text{diffusion}}$, and $H_{\text{integration}}$ each degrade the reproduction of frequencies below $f_{\text{Nyquist}}$ by a low-pass filtering effect, which may be characterized as $MTF_{\text{offchip-optics}}$, $MTF_{\text{onchip-optics}}$ $MTF_{\text{diffusion}}$, and $MTF_{\text{integration}}$, respectively [6], [45]. Since empirical characterization methods such as MTF measurements and laser spot scanning measure the combined effects of $H_{\text{onchip-optics}}$, $H_{\text{diffusion}}$, and $H_{\text{integration}}$, we define cross-responsivity of an imaging array as†

$$H_{\text{cross-resp}}(f) = H_{\text{onchip-optics}} \cdot H_{\text{diffusion}} \cdot H_{\text{integration}} \quad \text{for } f < f_{\text{Nyquist}}$$  \hspace{1cm} (4.1)

Fig. 4.2 illustrates the measurement setup for an empirical characterization of internal cross-responsivity of an image sensor. A Ne-He laser ($\lambda = 633$ nm) coupled to a thin core optical fiber (4.0 $\mu$m) with a pigtail style fiber focuser was employed for the experiment. The laser beam is focused onto a single pixel area and the generated photocarriers diffuse in all direction. The collection outcome of the photocarrier diffusion can be extracted from the sensor output. For simplicity, it is assumed that the measurement can be taken in the linear region of sensor operation with a proper selection of the integration time and the intensity of the illumination spot.

† Note that cross-responsivity is the 2D linear response of an imaging array from single pixel excitation. The DC gain of cross-responsivity as defined in (1) represents the quantum efficiency.
Fig. 4.1  A simple linear model of CMOS image sensor.

Fig. 4.2  The measurement setup for cross-responsivity PSF. Ne-He laser ($\lambda = 633$ nm) is coupled to a thin core optical fiber (4.0 $\mu$m) with a pigtail style fiber focuser.
As previously noted, a raster spot-scanning technique utilizing a relatively small spot (0.30 ~ 2.0 µm) has been effectively used in the investigation of sub-pixel photoresponse and lateral crosstalk [23]–[25]. With the spot-scanning technique, only the scanning field needs to be adjusted to accommodate different pixel sizes and shapes. Yet, the single spot illumination requires a much simpler setup and can be easily repeated on different pixels to verify measurement consistency. The circular shape of the spot, however, prevents the full coverage of corner areas of a square pixel; in addition, the non-uniform laser spot profile must be taken into account when using a single large spot to illuminate a pixel so that the surrounding pixels are not in order to prevent even slight illumination of surrounding pixels.

The profile of the employed laser spot was measured with a beam analyzer and was found to approximately Gaussian with standard deviation of about 0.85 µm in both XY dimensions. The spot hence offers 2-sigma diameter of 3.4 µm (i.e. 13.5% of the center intensity along the diameter) and 3-sigma diameter of 5.1 µm (i.e. 1.1% of center intensity).

Table 4.1 provides some information about four CMOS image sensors used in the experiment. For the larger pixels of ICDWTST4 and ICCWTCYC, the spot size was increased by adjusting the working distance with a micro-translator until the spot size was slightly below the size of each pixel. Upon adjusting the spot size, the integration time was also adjusted on each image sensor so that the illuminated pixel yielded 90% to 95% of the saturation output level. This was done to ensure a high signal-to-noise ratio while avoiding pixel blooming. An optical table was used to minimize the effects of vibration.
Fig. 4.3 Examples of CMOS active pixel sensors layout. (a) 22 µm x 22 µm APS in a standard 0.35 µm CMOS technology and (b) 10 µm x 10 µm APS in a standard 0.50 µm CMOS technology. The rectangular outline (in diagonal stripes) marks the photodiode area in each pixel. Each circle (in solid dark line) illustrates the spot size whose diameter is identical to the side length of each pixel. The dark regions in each layout represent metal interconnects in the pixel.

Fig. 4.4 Measured cross-responsivity PSF obtained from four CMOS image sensors. The measured cross-responsivity PSF are each scaled to provide a blur model with unity DC gain (i.e. \( \sum a_i = 1 \)).
Table 4.1 Some Information about the four CMOS image sensors in Fig. 4.4.

<table>
<thead>
<tr>
<th>Model</th>
<th>Technology</th>
<th>Pixel Pitch</th>
<th>Resolution</th>
<th>Fill Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>IBIS4 (FillFactory)</td>
<td>n/a (VDD=5.8V)</td>
<td>7.0 μm</td>
<td>1280x1024</td>
<td>60 %</td>
</tr>
<tr>
<td>ZR32112 (Zoran)</td>
<td>0.25 μm</td>
<td>7.5 μm</td>
<td>1280x1024</td>
<td>51 %</td>
</tr>
<tr>
<td>ICDWTST4 (custom)</td>
<td>TSMC 0.35 μm</td>
<td>22 μm</td>
<td>104x104</td>
<td>41 %</td>
</tr>
<tr>
<td>ICCWTCYC (custom)</td>
<td>HP 0.50 μm</td>
<td>30 μm</td>
<td>64x64</td>
<td>60 %</td>
</tr>
</tbody>
</table>

The measured cross-responsivity PSFs are shown in Fig. 4.4. The measured cross-responsivity PSF were each scaled to provide a blur model with unity DC gain (i.e. $\Sigma a_{ij} = 1$). The percentage of photocarriers participating in lateral crosstalk can be inferred from the center value of the cross-responsivity PSF; for example, the center value of 73.6% for the commercial IBIS4 sensor indicates that 26.4% of the photocarriers excited from the center pixel manifest in the neighboring pixels. Examining the results of Fig. 4.4, it can be seen that the smaller pixels generally show higher lateral crosstalk as expected. An exception is found with ZR32112, which exhibits an anisotropic PSF with a relatively little lateral crosstalk. Several fabrication process characteristics can significantly vary lateral photoresponse from one technology to another, including the epitaxial layer thickness [26], [29], the epitaxial and substrate doping [26], [42] surface recombination and mobility degradation near the Si-SiO$_2$ interface [24], as well as the optical characteristics of the technology [39], [43], [44].
From Fig. 4.4, it is also evident that some cross-responsivity PSFs are slightly asymmetric. While bulk diffusion is practically isotropic for imaging applications, the presence of reverse biased junctions and gate electric fields in a pixel structure can significantly deter lateral diffusion. The asymmetry of lateral photoresponse in CMOS active pixel sensors has been also demonstrated with sub-pixel photoresponse characterizations performed with the spot-scanning technique [39]–[42].

The loss of photocarriers to lateral crosstalk directly translates to a loss of image contrast as illustrated in Fig. 4.5, which shows a test image consisting of several black stripes on a white background convolved with the cross-responsivity PSF of IBIS4. Fig. 4.5 also shows the result of applying the inverse filter on the blurred image, i.e.

\[ G_{\text{inverse}} = H_{\text{cross-responsivity}}^{-1} \]  \hspace{1cm} (4.2)
Fig. 4.5 A test image consisting of several black stripes on a white background convolved with the cross-responsivity PSF of IBIS4. The resulting loss of contrast characterized by MTF degradation is found to be about 29% – slightly higher than 26.4% indicated by the cross-responsivity PSF – as the photocarriers lost to lateral crosstalk manifest in the neighboring pixels to further degrade the contrast gradient.

In the next section, the conditions necessary for an effective restoration of images from the blurring effects of lateral crosstalk are discussed. Several well-known linear deconvolution filters are also demonstrated, illustrating some of the tradeoffs in designing a linear deblurring operation for image sensor applications.
4.2 Comparison of Deblurring Filters

For exponential or Gaussian type blurring, the Fourier transform of the blur model shows a frequency response that is non-zero at all frequencies, implying that the blur process preserves all frequency components of the original image.† In practical imaging systems with finite range of operating frequencies, it can be generally stated that a blur process must preserve all significant frequency components of an image with respect to the system’s minimum noise limit in order for the blur process to be effectively reversible. Several investigations of lateral photoresponse in modern CMOS image sensors have revealed that the characteristic distance of spatial decay of lateral photoresponse is generally comparable to the typical pixel sizes (i.e. 5 ~ 10 µm) [39]–[41]. This is also demonstrated by the 3x3 cross-responsivity PSFs shown in Fig. 4.4. From the observation that the spatial extent of lateral photoresponse is comparable to the period of the sampling operation, it can be inferred that $H_{\text{diffusion}}$ likely spans the whole frequency spectrum of the CMOS imaging array.

Fig. 4.6 shows the result of inverse filtering on a captured image from IBIS4. The improvement of contrast from the inverse filtering on an actual image output of IBIS4 in Fig. 4.6 is considerably smaller than the previous case with simulated lateral crosstalk (Fig. 4.5). This follows from the observation that the images captured by an image sensor undergo a number of other blurring processes in addition to the lateral crosstalk such as those arising from the image sensor optics (i.e. $H_{\text{optics}}$). Fig. 4.6 also reveals that inverse filtering leads to a slight amplification of high-frequency noise (e.g. fixed pattern noise). It is useful to note an inverse filter can employ a very large high-frequency gain, which can significantly amplify high-frequency noise [47], [48]

† A local average filter, on the other hand, has frequency response of the form $\sin(f)/f$ and demonstrates a non-invertible blur process [47].
and provide an excessive enhancement some bright pixels – causing their intensities to be clipped
the highest gray-level – thus resulting in a loss of image content [49]. Fig. 4.7 illustrates the
effect of such an excessive high-frequency enhancement in a deblurring operation.

![Captured Image](image1)

![Inverse Filtering](image2)

**MTF = 0.15**

**MTF = 0.22**

Fig. 4.6 Inverse filtering on captured images of IBIS4. The figure on the right is the captured and the one on the
left is the result of inverse filtering. The MTF values were calculated using the maximum and minimum output
levels of the image sensor.

In order to demonstrate linear deblurring filters with constrained high-frequency gain, Wiener
decovolution and constrained least square restoration with smoothness constraint (hereon
referred to as CLSR) were selected. The details of these filters can be found in digital image
processing texts such as Castleman [47], and Gonzalez and Woods [48]. Both Wiener
deconvolution and CLSR constrain their high-frequency gain based on an estimation of noise power from the input image. Given that the observed noise predominantly originates from the sensor itself, the noise estimation was extracted from the measured noise characteristics of the employed image sensors. Fig. 4.8 summarizes the noise measurements performed on IBIS4 and ZR32112. The photoresponse non-uniformity (PRNU) was measured at various illumination intensities and at various integration times (with frame averaging to remove random noise). The random noise was measured by subtracting the dark frame from each captured frame and averaging the noise power over one hundred frames.

![Fig. 4.7 An illustration of excessive high-frequency gain in a deblurring operation. (a) captured images from ZR32112, (b) inverse filtering, (c) a deblurring operation with excessive high-frequency gain.](image-url)
Fig. 4.8 Photoresponse non-uniformity (PRNU) measurements on IBIS4 and ZR32112 using monochromatic illumination source ($\lambda = 540$ nm) and an integrating sphere. The solid dots were obtained by varying integration time at constant illumination; the hollow dots were obtained by varying illumination intensity with a fixed integration time. Both fixed pattern noise (FPN) and random noise were measured in dark with integration time of 30 ms.
In both sensors PRNU was significantly higher than the observed fixed pattern noise (FPN) and random noise. Fig. 4.8 illustrates that PRNU varies linearly with the average intensity of the output image and behaves essentially independent of both the incident illumination intensity and the integration time.† As the noise characteristics in both sensors were dominated by PRNU, the noise estimations in Wiener deconvolution and CLSR were obtained from the measured values of PRNU. The highest value of the measured PRNU was used for the noise power estimation, as the noise levels in the employed CMOS image sensors were found to be rather low for an illustrative demonstration of the noise adaptive filters.

Fig. 4.9 compares the results of inverse filtering, Wiener deconvolution, and CLSR on various captured images of IBIS4. Due to the low level of noise in IBIS4 (< 3.5%), the three filters produce comparable results. The constrained high-frequency gain in Wiener deconvolution and CLSR lead to a slightly compromised deblurring as indicated by MTF values calculated from the deblurred images. The amplification of high-frequency noise in Wiener deconvolution and CLSR is also slightly lower as expected. Other linear deblurring filters offer similar trade-offs between the image sharpness and the moderation of high-frequency amplification. A quantitative evaluation of various trade-offs can lead to a deblurring filter that is optimized for a given imaging system. In addition, in cases where a suitable compromise between the trade-offs cannot be achieved with linear deblurring operation, adaptive or non-linear methods can be considered to further improve on the restoration.

† This suggests that for both sensors, PRNU originate dominantly from the mismatches in readout (and amplifying) circuits rather than from the mismatches in photosensitive device (e.g. photodiode, photogate), which may lead to a more noticeable dependence on the incident illumination level or the integration time.
4.3 Discussions

In presented illustrations of deblurring operation with the blur model of lateral crosstalk, the image sharpness of the incident image is recovered only partially as there are other blurring processes in a CMOS image sensor system. A comprehensive blur model of CMOS image sensor including the effects of the optics ($H_{\text{optics}}$) can lead to a more effective deblurring operation. In addition to the design of a deblurring solution, the characterization of various blurring effects would also reveal the aspects of the system that primarily constrain the final resolution of the image sensor.

The implementation of a deblurring operation can significantly vary depending on the final blur model of the system.\textsuperscript{†} As the ideal deblurring operation may be too costly to implement, a viable alternative may entail utilization of a sharpening filter devised with a trial-and-error method. Fig. 4.10 compares the results of inverse filtering, unsharp masking, and blur subtraction on the output images of IBIS4 [47], [48]. The result of blur subtraction demonstrates significant amount of high-frequency artifacts introduced by the sharpening filter. It can thus be appreciated that while a sharpening filter may be utilized to achieve similar effects as a restorative deblurring operation, a careful design is required in order to prevent the introduction of undesirable artifacts. This follows the observation that a sharpening filter may over compensate for the blurring effects of the CMOS image sensor.

\textsuperscript{†} The design methods for linear and non-linear adaptive deblurring filters and their suitable implementations can be found in digital image processing and filter design texts [47], [48], [51], [52].
Fig. 4.9 Comparison of inverse filtering, Wiener deconvolution, and CLSR on a test image captured with IBIS4.
Given a deblurring operation for the treatment of lateral crosstalk and other blurring processes, the improvement in geometric MTF following a pixel size reduction can have a higher impact on the effective resolution of the image sensor. Assuming two-dimensional array of identical rectangular pixels of width X and length Y, the geometric MTF of the sampling operation ($MTF_{\text{integration}}$) can be characterized as [6], [27]

$$MTF_{\text{integration}} = \text{sinc}(f_x X) \cdot \text{sinc}(f_y Y) \quad \text{for} \quad f_x X, f_y Y \ll 2$$  \hspace{1cm} (4.3)

where $f_x$ and $f_y$ are the spatial frequency of the intensity variation along the row and the column directions, respectively. Table 4.2 shows calculated values of $MTF_{\text{integration}}$ for several pixel sizes. The increase in resolving power with pixel size reduction can be appreciated from the observation that a higher spatial sampling frequency implies a more precise capture of the bright and the dark edges of an image. The results of Table 4.2 demonstrate that a significant improvement in the geometric MTF can be achieved with small reductions in pixel size. Fig. 4.11 illustrates the calculations in Table 4.2 with a series of simulated images.

The cross-responsivity measurement using focused laser significantly depends on the wavelength of the laser as the lateral diffusion of photocarriers ($H_{\text{diffusion}}$) depends on the absorption depth of the incident photons [39]–[41]. In the case of color image sensors employing color filter array (CFA), cross-responsivity can be characterized for each wavelength of the color filter to devise a space-variant blur model of the sensor.‡ For monochrome image sensors, cross-responsivity characterization using different wavelengths can provide a wavelength-dependent

‡ Alternatively, a group of pixels that form elementary color pattern unit in the employed CFA can be treated as a single “pixel” in order to construct a space-invariant blur model at a lower spatial resolution.
blur model from which a suitable one may be selected as the image sensor application may detail general wavelength spectrum of the lighting and the extent of deblurring required for optimal image sharpness.

Fig. 4.10  (a) Comparison of inverse filtering (i.e. deblurring) with two 3x3 local sharpening operators

Fig. 4.10  (a) Comparison of inverse filtering (i.e. deblurring) with two 3x3 local sharpening operators
Fig. 4.10  (b) Comparison of inverse filtering (i.e. deblurring) with two 3x3 local sharpening operators.
Table 4.2 Improvement of geometric MTF with pixel size reduction with respect to sinusoidal target with \( f_x = f_y = 1/16 \) µm (Nyquist frequency of the original 8 µm x 8 µm array).

<table>
<thead>
<tr>
<th>( X = Y )</th>
<th>( M_{TF_{integration}} )</th>
<th>( X = Y )</th>
<th>( M_{TF_{integration}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0 µm</td>
<td>0.64</td>
<td>5.0 µm</td>
<td>0.85</td>
</tr>
<tr>
<td>7.0 µm</td>
<td>0.71</td>
<td>4.0 µm</td>
<td>0.90</td>
</tr>
<tr>
<td>6.0 µm</td>
<td>0.78</td>
<td>3.0 µm</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Fig. 4.11 Illustration of the results in Table 4.2. (a), (b), and (c) illustrate contrast gain of the geometric MTF calculated in Table 4.21 for \( X = 8.0 \) µm, 6.0 µm, 4.0 µm, respectively. (d) is the result of 2x2 local average filtering on a test image shown in (e) which illustrates the sharpness gain in halving the pixel size (e.g. 8.0 µm to 4.0 µm).
4.4 Summary

A simple technique for an empirical characterization of lateral crosstalk was demonstrated (Fig. 4.2). The blurring effect of lateral crosstalk was characterized by the cross-responsivity PSF of an imaging array, which can be obtained from a single pixel excitation (Fig. 4.4). The inverse filtering, Wiener deconvolution, and constrained least square restoration with smoothness constraint were used to demonstrate deblurring lateral crosstalk (Fig. 4.9). The noise estimation in Wiener deconvolution and CLSR was obtained from the measured PRNU of the employed CMOS image sensors as it dominated the noise characteristics in the employed sensors (Fig. 4.8). The comparison of linear deconvolution filters led to a brief illustration of the tradeoffs in image sharpness and the high-frequency noise effects. As the implementation of a comprehensive deblurring solution may afford designers to consider further pixel size reduction, simple calculations on pixel size versus geometric MTF were provided to facilitate further discussions.
Chapter 5

Conclusions

As several aspects of technology scaling introduce additional challenges to the CMOS image sensor design, comprehensive models of the CMOS based pixels are necessary in order to translate given design objectives to proper choices of technology, pixel architecture, array structure, design layout, etc. Toward this end, improved analyses of CMOS photodiodes and photodiode arrays were presented with emphasis on the significance of surface recombination, mobility degradation along the Si-SiO$_2$ interface, epitaxial thickness, and epitaxial-substrate junction.

Chapter 2 presented one-dimensional analysis of a single-junction photodiode for a standard CMOS technology. As the intention of the analysis is to serve as a basis of a compact model of photodiode, relatively simple expressions with the adoption of two empirically fitted parameters were discussed. The presented model of quantum efficiency demonstrated a reasonable agreement with measured spectral response of n$^+$$p_{epi}$ photodiode fabricated on a standard 0.35 $\mu$m CMOS technology.

The presented analysis, however, is based on (1) the assumptions of low-injection, and (2) the approximation of the epitaxial-substrate interface with an ideal carrier sink. At very high illumination intensities, these approximations can fail and the presented analysis may significantly overestimate the actual photocurrent, as it does not account for the increased bulk recombination in both the epitaxial layer and the substrate and the deviation from the ideal carrier sink characteristics at the high-low junction [15]. Therefore, the presented analysis of
photocurrent and the resulting compact model may significantly overestimate photocurrent at very high-illumination intensities.

The presented compact model needs to be further evaluated with photodiodes fabricated on various CMOS processes in order to identify the limitations of the model with respect to the assumed photodiode structure and the range of operating conditions. In order to evaluate the presented model with graded-junction photodiodes such as nwell-p_epi, the doping profile of the photodiode junction should be extracted or otherwise obtained in order to assess the suitability of the presented model. A graded-junction may lead to reverse-bias sensitivity of the photodiode, which may require additional considerations in the usage of the presented model.

An improved model of optical reflections in CMOS overlayers is also necessary in order to better account for the observed interference pattern in the spectral response of a CMOS photodiode. The information regarding the composition and the thickness of each CMOS passivation/insulation layer is not generally reported; yet, a multi-film optical model based on the actual structure of the passivation/insulation stack of the considered process may provide a better account of the optical effects in the fabricated photodiodes.

In Chapter 3, the presented three-dimensional analysis of edge-effect demonstrated a reasonable agreement with the measured data despite the assumptions of closely spaced photodiodes and shallow junction depth (Fig. 3.5). Yet, the presented analytical approximation neglects surface recombination and mobility degradation along the Si-SiO₂ interface, and, hence empirical characterization of edge-effect was recommended as a more reliable approach. An empirical characterization method utilizing a set of linear photodiode arrays was illustrated (Fig. 3.1). The plot of photocurrent versus photodiode separation distance revealed a relatively smooth exponential-like function.
While this method provides a simple and reliable approach, a disadvantage is that a significant die area is required to implement the test structure, and the test structure needs to be repeated for each type of photodiode. In the illustrated experiment with $n^+-p_{epi}$ photodiodes, approximately $3\,\text{mm} \times 1\,\text{mm}$ die area was required to implement the test structure. A question also arises as to what perimeter of photodiode strip (or more accurately perimeter-to-area ratio) is sufficient to accurately characterize two-dimensional edge-effect, or how the observed edge-effect will vary as the perimeter is decreased (i.e. from $30\,\mu\text{m} \times 5\,\mu\text{m}$ used in the edge-effect experiment discussed in Chapter 3). While it is certain that two-dimensional edge-effect can be characterized by a sufficiently long photodiode strip, a further examination of the dependence on the photodiode strip structure is required in order to design a test structure with an efficient use of die area.

The simple linear extension of the two-dimensional edge-effect to the case of two-dimensional rectangular array demonstrated in (3.20) proved to be only useful in the limited case where the photodiodes are closely spaced in relation to its dimensions (Fig. 3.7). Hence, better methods of utilizing the two-dimensional characterization of edge-effect for the case of two-dimensional rectangular photodiode should be explored; alternatively, an innovative design of two-dimensional rectangular photodiode arrays may lead to a direct characterization of three-dimensional edge-effect characterization structure with acceptable area requirements.

In Chapter 4, an empirical characterization of lateral crosstalk in CMOS image sensors was demonstrated with a relatively simple measurement setup (Fig. 4.2). The blurring effect of lateral crosstalk was characterized with cross-responsivity PSF obtained from a single pixel excitation (Fig. 4.4). The measurement revealed $3 \times 3$ cross-responsivity PSF consistent with recently reported laser excitation experiments with CMOS image sensors [39]–[42].
While the single laser spot illumination requires a much simpler setup than raster scanning with a smaller spot, this method relies on the general observation that the peripheral regions of a pixel employing an active readout circuitry are typically occupied by metal interconnect lines. This observation may not hold true in certain pixel designs rendering this method unsuitable for the lateral crosstalk characterization. For rectangular pixels, the single spot illumination may also fail to illuminate a significant portion of the pixel area, and require a more flexible method such as the raster scanning.

A demonstration of several linear deblurring filters illustrated the tradeoffs in image sharpness and the high-frequency noise effects. The deblurring operation based on the blurring effects of lateral crosstalk alone led to only slight improvement in the image sharpness. It was concluded that other blurring effects such as those originating from optics should be incorporated into the blur model in order to provide a more effective, comprehensive deblurring solution.

As an alternative to the systematic development of the blur model for a given CMOS image sensor, simple local sharpening operators should also be explored as another possible solution. While an implementation of a deblurring solution may require a very large window size or an expensive implementation of a recursive equation, local sharpening operators can be designed with specific implementation in mind. While the restriction of the window size may lend local sharpening operators to undesirable high-frequency artifacts (Fig. 4.7), their cost-to-performance trade-off may offer practical alternatives to deblurring operations.

While the implementation of a deblurring solution may afford designers to consider further pixel size reduction for higher resolution and/or smaller die area, further considerations of other image sensor parameters including the optics, noise and dynamic range of the pixel, and the various implementation costs are required to determine an optimal pixel size of an image sensor.
[53]. Further investigations of lateral crosstalk and other blurring processes in CMOS image sensors are needed in order to analyze the cost-to-performance trade-off in implementing a comprehensive deblurring solution, which may enable an effective deployment of very small pixel sizes.
Appendix A

One-Dimensional Analysis of Photodiode

An extension of the classical one-dimensional analyses provided by van de Wiele [15] is provided as Appendix A. Consider a photodiode structure shown in Fig. A.1.

![Fig. A.1 An example of one-dimensional photodiode structure. Note that z-axis represents the dimension along the depth of photodiode. This is to use same notations as the three-dimensional analysis presented in Chapter 3.](image)

It is assumed that the quasi-neutral regions \((0, z_n)\) and \((z_p, L)\) are uniformly doped by \(N_D\) of donor and \(N_A\) of acceptor impurities, respectively. The photodiode can then be divided into two quasi-neutral regions separated by a depletion region \((z_n, z_p)\). For each quasi-neutral region we assume that the density of excess carriers remains small compared to the equilibrium majority carrier density i.e. the case of low injection of excess carriers is considered. This assumption implies that the majority carrier concentration is almost equal to its equilibrium value, and the electrostatic potential, \(\Psi(z)\), may be considered as constant and equal to its equilibrium value, which is taken as zero.
\[
\psi(z) = 0 \quad \text{for } 0 < z < z_n \tag{A.1}
\]

Under non-equilibrium conditions a small non-zero electric field exists within the quasi-neutral region. Yet, this small electric field is small compared to the potential across the depletion region and is generally neglected in quasi-equilibrium approximation. The quasi-neutral region \((z_p, L)\) may be treated in a similar way:

\[
\psi(z) = 0 \quad \text{for } z_n < z < L \tag{A.2}
\]

The built-in potential of an abrupt junction is given by [7]

\[
V_{bi} = (kT/q) \ln(n_i^2/N_A N_D) \tag{A.3}
\]

Within the interval \((z_n, z_p)\) the depletion approximation is used i.e. the density of free carriers is supposed negligible compared to the net density of ionized impurities. From the continuity equation and the charge neutrality condition, the boundaries of the depletion region are given by

\[
z_n = z_j - \frac{2 \varepsilon N_A (V_A - V_{bi})}{qN_D (N_A + N_D)} \tag{A.4}
\]

\[
z_p = z_j + \frac{2 \varepsilon N_D (V_{bi} - V_A)}{qN_A (N_D + N_A)} \tag{A.5}
\]
where $\varepsilon$ is the permittivity of the substrate, $N_D (N_A)$ is the density of ionized donor (acceptor) impurities, and $z_j$ is the metallurgical junction depth.

**A.1 Analysis of Surface Region ($0, z_n$)**

Neglecting the presence of the electric field in the surface region ($0, z_n$) and assuming steady-state condition, the continuity equation becomes,

$$\frac{d^2 \hat{p}}{dz^2} - \frac{\hat{p}}{L_n} + \frac{G(z)}{D_n} = 0$$

(A.8)

The following boundary conditions are imposed for the surface region:

at $z = 0$: $D_n \frac{d\hat{p}}{dz} \bigg|_{z=0} = v_s \hat{p}(0)$  \hspace{1cm} (A.9)

at $z = z_n$: $\hat{p}(z_n) = 0$  \hspace{1cm} (A.10)

The first condition accounts for the surface recombination as characterized by the surface recombination velocity, $v_s$. The second condition is the classical Boltzmann equation valid under low injection conditions. Solving the continuity equation in (A.8) with the boundary conditions (A.9) and (A.10) yields the following [15]:

79
\[
\hat{p}_{ph}(z) = \frac{G_0 L_p^2}{D_p \left(1 - \frac{L_p^2}{L_{op}^2}\right)} \left[ e^{\frac{z}{L_p}} + \frac{L_p \left(s + \frac{D_p}{L_{op}}\right) \sinh\left(\frac{z - z_n}{L_p}\right) - e^{\frac{z_n}{L_p}} \left[ s L_p \sinh\left(\frac{z}{L_p}\right) + D_p \cosh\left(\frac{z}{L_p}\right)\right]}{s L_p \sinh\left(\frac{z}{L_p}\right) + D_p \cosh\left(\frac{z}{L_p}\right)} \right]
\]

(A.11)

where \( G_0 \) is the peak photogeneration rate at \( z = 0 \) given by \( G(x) = \alpha F_0 e^{-\alpha x} = G_0 e^{-\alpha x} \). The resulting diffusion current density, \( J_{ph}(z) \), is given by

\[
J_{ph}(z) = \frac{q G_0 L_p^2}{D_p \left(1 - \frac{L_p^2}{L_{op}^2}\right) L_{op}} \left[ e^{\frac{z_n}{L_{op}}} \left( s + \frac{D_p}{L_{op}} \right) \cosh\left(\frac{z - z_n}{L_p}\right) - e^{\frac{z_n}{L_p}} \left[ s L_p \cosh\left(\frac{z}{L_p}\right) + D_p \sinh\left(\frac{z}{L_p}\right)\right] \right]
\]

(A.12)

At \( z = z_n \), the photocurrent expression of (A.12) reduces to

\[
J_{ph}(z_n) = \frac{q G_0 L_p^2}{D_p \left(1 - \frac{L_p^2}{L_{op}^2}\right) L_{op}} \left[ e^{\frac{z_n}{L_{op}}} \left( s + \frac{D_p}{L_{op}} \right) - e^{\frac{z_n}{L_p}} \left[ s L_p \cosh\left(\frac{z_n}{L_p}\right) + D_p \sinh\left(\frac{z_n}{L_p}\right)\right] \right]
\]

(A.13)

Note that even when no light in incident upon the photodiode, a leakage current through the diode still exists due to the thermal excitation of mobile carriers. The resulting current across the
photodiode under no illumination is called the dark current, and it represents the noise floor of the photodiode operation.

The performance of photodiode is generally expressed in term of the quantum efficiency, $\eta$, which is defined as the ratio of the number of photogenerated carriers contributing to the collected photocarriers (i.e. photocurrent) to the number of incident photons. For the quasi-neutral region $(0, z_n)$ a partial internal quantum efficiency, $\eta_{\text{surface}}$, may be defined as

$$\eta_{\text{surface}} = \frac{J_{ph}(z_n)}{qG_\alpha L_{op}}$$ \hspace{1cm} (A.14)

Its value is

$$\eta_{\text{surface}} = \frac{L_p^2}{L_{op}} \left( 1 - \frac{L_p^2}{L_{op}^2} \right) \left\{ \frac{e^{\frac{z_n}{L_{op}}} \left[ s \left( 1 + \frac{D_p}{L_{op}} \right) - e^{\frac{z_n}{L_{op}}} \left[ sL_p \cosh \left( \frac{z_n}{L_p} \right) + D_p \sinh \left( \frac{z_n}{L_p} \right) \right] \right]}{sL_p \sinh \left( \frac{z_n}{L_p} \right) + D_p \cosh \left( \frac{z_n}{L_p} \right)} \right\}$$ \hspace{1cm} (A.15)

For large values of the minority carrier lifetime, and hence the diffusion length, $L_p$, the partial quantum efficiency becomes,

$$\eta_{\text{surface}} = \frac{s L_{op} \left( 1 - e^{\frac{z_n}{L_{op}}} \right) + D_p}{s z_n + D_p} - e^{\frac{z_n}{L_{op}}}$$ \hspace{1cm} (A.16)
Furthermore, for a zero recombination velocity, \( s \), the so-called ideal case is obtained

\[
\eta_{\text{surface}} = 1 - e^{\frac{z_n}{L_p}}
\]  

(A.17)

In this case the optimum value of \( \eta_{\text{surface}} \) is obtained since all photogenerated minority carriers contribute to the photocurrent.

**A.2 Analysis of Depletion Region \((z_n, z_p)\)**

For photoresponse in the depletion region of a photodiode, it is generally assumed that all optically generated carriers effectively contribute to the photocurrent since the recombination in the depletion region is considered as negligible [15]. The contribution of the entire depletion region to the photocurrent is then,

\[
J_{\text{ph}} = G_0 L_{op} \left( e^{\frac{z_n}{L_p}} - e^{\frac{z_p}{L_p}} \right)
\]  

(A.18)

The partial quantum efficiency of the photodiode depletion region, \( \eta_{\text{DR}} \), is then,

\[
\eta_{\text{DR}} = e^{\frac{z_n}{L_p}} - e^{\frac{z_p}{L_p}}
\]  

(A.19)
A.3 Analysis of Substrate Region \((z_p, L)\)

For the substrate region, a number of different conditions may prevail at the backside of the substrate \((z = L)\) depending on the technology employed. The solutions based on boundary conditions (1) and (2) below have been provided in [15]. The solutions based on boundary conditions (3) and (4) have been added here as a reference for the reader.

**Boundary Condition 1: Finite Substrates with Perfectly Collecting Backside** [15]

\[
\hat{n}(z_p) = 0 \quad \hat{n}(L) = 0 \tag{A.20}
\]

**Boundary Condition 2: Semi-infinite Substrates** [15]

\[
\hat{n}(z_p) = 0 \quad L \gg L_n \tag{A.21}
\]

**Boundary Condition 3: Finite Substrates with Minority Carrier Reflecting Backside**

\[
\hat{n}(z_p) = 0 \quad \left. \frac{d\hat{n}}{dz} \right|_{z=L} = 0 \tag{A.22}
\]

**Boundary Condition 4: Epitaxial Layer atop more Heavily Doped Substrate** (e.g. standard CMOS)

\[
\hat{n}(z_p) = 0 \quad \left. \frac{d\hat{n}}{dz} \right|_{z=L} = \Phi_x \tag{A.23}
\]
Note that $\Phi_s$ in the boundary condition (A.23) denotes the flux of photogenerated minority carriers injected into the epitaxial layer from the substrate due to the high-low doped junction. This case is illustrated in Chapter 2 and is not repeated here.

Solving to the Continuity equation in (A.8) with above boundary conditions produce the following solutions [15]:

Solution 1: Finite Substrates with Perfectly Collecting Backside [15]

\[
\hat{n}_{ph}(z) = -\frac{G_o L_n^2}{D_n \left( l - \frac{L_p^2}{L_{op}^2} \right)} \left\{ e^{-\frac{z}{L_{op}}} \sinh \left( \frac{z - z_p}{L_n} \right) + e^{-\frac{L - z}{L_{op}}} \sinh \left( \frac{L_z}{L_n} \right) \right\}
\]

(A.24)

\[
J_{ph}(z) = \frac{qG_o L_n^2}{\left( l - \frac{L_p^2}{L_{op}^2} \right) L_{op}} \left\{ e^{-\frac{z}{L_{op}}} \cosh \left( \frac{z - z_p}{L_n} \right) - e^{-\frac{L - z}{L_{op}}} \cosh \left( \frac{L_z}{L_n} \right) \right\}
\]

(A.25)

The partial quantum efficiency, $\eta_{\text{substrate}}$, may be defined as

\[
\eta_{\text{substrate}} = \frac{J_{ph}(z_p)}{-qG_o L_{op}}
\]

(A.26)

Hence,
\[ \eta_{\text{substrate}} = \frac{L_n^2}{L_{op}^2} \left( \frac{\frac{L_n}{z_p}}{L_{op}} - 1 \right) \left( e^{\frac{L}{L_{op}}} - e^{\frac{L - L_p}{L_{op}}} \cosh \left( \frac{L - L_p}{L_n} \right) \right) \]  

(A.27)

For large values of the minority carrier lifetime and hence diffusion length, the partial quantum efficiency simplifies to

\[ \eta_{\text{substrate}} = e^{\frac{-z_p}{L_{op}}} \left( \frac{L_{op}}{L - z_p} \right) \]  

(A.28)

Solution 2: Semi-infinite Substrates [15]

The solution for semi-infinite substrate is simply a limiting case of the finite substrate where \( L \to \infty \). Therefore,

\[ \hat{n}_{ph}(z) = \frac{-G_\alpha L_n^2 e^{\frac{z}{L_{op}}}}{D_e \left( 1 - \frac{L_p^2}{L_{op}^2} \right)} \]  

(A.29)
\[ J_{ph}(z) = \frac{qG_0 \frac{L_n^2}{L_{op}} e^{\frac{-z}{L_{op}}}}{\left(1 - \frac{L_n^2}{L_{op}^2}\right)} \]  

(A.30)

Applying (A.26)

\[ \eta_{\text{substrate}} = \frac{L_n^2}{L_{op}} e^{\frac{-z}{L_{op}}} \]

(A.31)

The ideal case occurs for a semi-infinite substrate and an infinite lifetime of the carriers

\[ \eta_{\text{substrate}} = e^{\frac{-z}{L_{op}}} \]  

(A.32)

Solution 3: Finite Substrates with Minority Carrier Reflecting Backside

\[ n_{ph}(z) = K_3 e^{\frac{z}{L_n}} + K_2 e^{\frac{-z}{L_n}} + K_1 e^{\frac{-z}{L_{op}}} \]  

(A.33)

where

\[ K_1 = -\frac{G_0}{D_n} \left( \frac{L_{op}^2 L_n}{L_n^2 - L_{op}^2} \right) \]  

(A.34)
\[ K_2 = K_4 \left( e^{-z_p \left( \frac{1}{L_{op}} - \frac{1}{L_n} \right)} + \frac{L_n}{L_{op}} e^{\frac{z_p}{L_{op}}} \right) \]

\[ K_3 = -K_2 e^{\frac{-z_p}{L_n}} - K_4 e^{\frac{-z_p}{L_{op}}} \]

\[ J_{ph}(z) = -qD_e \left( \frac{K_1}{L_n} e^{\frac{z}{L_n}} - \frac{K_2}{L_n} e^{\frac{-z}{L_n}} - \frac{K_4}{L_{op}} e^{\frac{-z}{L_{op}}} \right) \]

Applying (A.26)

\[ \eta_{subrate} = \frac{D_n}{G_o L_{op}} \left( \frac{K_1}{L_n} e^{\frac{z_p}{L_n}} - \frac{K_2}{L_n} e^{\frac{-z_p}{L_n}} - \frac{K_4}{L_{op}} e^{\frac{-z_p}{L_{op}}} \right) \]
Appendix B

Simulation of Substrate Injection of Photocarriers

TITLE Effects of high-low junction of epi-substrate + interface on the steady-state photoresponse

COMMENT + Updated parameters for 0.35um technology.
+ by Ji-Soo Lee
+ on 02/06/2001

COMMENT Variable declarations
ASSIGN NAME=EPITHCK N.VALUE=3.0
ASSIGN NAME=SUBTHCK N.VALUE=24.0
ASSIGN NAME=DOPSUB N.VALUE=2.0E15
ASSIGN NAME=DOPNPLUS N.VALUE=1.0E20
ASSIGN NAME=DOPPPLUS N.VALUE=1.0E20
ASSIGN NAME=JUNCDEP N.VALUE=0.10
ASSIGN NAME=APPVOL N.VALUE=1.0

LOOP STEPS=4
ASSIGN NAME=I N.VALUE=(1,2,3,4)
ASSIGN NAME=J N.VALUE=(0.01,10,100,1000)
ASSIGN NAME=DOPEPI N.VALUE=@DOPSUB/@J

COMMENT Create an initial simulation mesh
MESH REC
X.MESH X.MAX=0.5 H1=0.05
Y.MESH Y.MAX=@EPITHCK H1=0.005 H2=0.05
Y.MESH Y.MAX=@SUBTHCK H1=0.05 H2=1.00

COMMENT Region
REGION NUM=1 Y.MAX=@EPITHCK SILICON
REGION NUM=2 Y.MIN=@EPITHCK SILICON

COMMENT Specify Electrodes
ELECTR NAME=Collector TOP
ELECTR NAME=Substrate BOTTOM

COMMENT Specify impurity profiles
PROFILE P-TYPE N.PEAK=@DOPEPI UNIF OUT.FILE=ds
PROFILE P-TYPE N.PEAK=@DOPSUB UNIF
+ Y.MIN=@EPITHCK
PROFILE N-TYPE N.PEAK=@DOPNPLUS
+ Y.MIN=0 Y.JUNC=@JUNCDEP XY.RAT=0.75

COMMENT Set minority carrier lifetimes based on empirically obtained diffusion lengths
MATERIAL REGION=1 TAUN0=2.35e-5
MATERIAL REGION=2 TAUN0=2.35e-7
MOBILITY SILICON MUN0=1.5E3

COMMENT Refine the mesh with doping regrids
REGRID DOPING LOG RAT=1 SMOOTH=1 IN.FILE=ds
+ OUT.FILE="mesh"@I""
COMMENT Display the simulation mesh
PLOT.2D TITLE="Mesh" FILL GRID Y.MAX=@SUBTHCK
PLOT.1D TITLE="Doping Concentration"
PLOT.1D TITLE="Doping Concentration"
  + DOP LOG X.START=0.25 X.END=0.25 Y.START=0 Y.END=@SUBTHCK

COMMENT Specify physical models to use
MODELS SRH AUGER FLDMOB BGN

COMMENT Calculate a steady state solution with reverse bias of 1.0 Volts. Perform a zero carrier solution to use as the initial guess for the two carrier solution.
SYMB GUMMEL CARRIERS=0
METHOD DAMPED ICCG
SOLVE V(Collector)=@APPVOL V(Substrate)=0

COMMENT Specify incident illumination via characteristic absorption distance and photon flux density.
ASSIGN NAME=Y.CHAR N.VALUE=6.0
ASSIGN NAME=FLUX N.VALUE=1E17

COMMENT Illuminate
PHOTOGEN A3=1E4*@FLUX/@Y.CHAR A4=-1/@Y.CHAR

COMMENT Open a log file for storing the terminal data
LOG OUT.FILE="log"@I"

COMMENT Switch to Newton and two carriers and solve for the steady state reverse bias solution.
SYMBOL NEWTON CARRIERS=2
SOLVE V(Collector)=@APPVOL V(Substrate)=0

COMMENT Plot the excess minority carrier density
PLOT.1D ELECTRON LOG
  + X.START=0.25 X.END=0.25
  + Y.START=@EPITHCK Y.END=@EPITHCK+5.0
  + OUT.FILE="exelec"@I".txt"

PLOT.1D J.ELEC
  + X.START=0.25 X.END=0.25
  + Y.START=@EPITHCK Y.END=@EPITHCK+1.0
  + OUT.FILE="current"@I".txt"

L.END
Appendix C

Photodiode Design with Substrate Openings

The readout amplifier for CCD detectors and CMOS active pixel sensors is usually a floating-diffusion device sensed by a source-follower; the capacitance of the device determines the conversion gain. For CMOS active pixel sensors (APS), the capacitance of the conversion node is typically dominated by the junction capacitance of the photodiode (Fig. C.1). In order to improve the conversion gain, it is thus desirable to minimize this capacitance while maintaining the collection efficiency. The reset noise of active pixel sensor, which is generally modeled as $kTC$ noise, is also lowered with the reduction of the photodiode capacitance [12].

![Schematic of photodiode-type active pixel sensor (APS).](image)

A shape of photodiode that offers lower capacitance at a minimal loss of collection efficiency is a photodiode shape with one or more substrate openings as shown in Fig. C.2. Similar structures have been proposed previously [54]. Employing one or more substrate openings in the...
photodiode design for APS reduces area of lateral junction although it also introduces additional side-wall junctions; hence, the size of the opening needs to be large enough to reduce the net capacitance. Since the signal output of an APS is proportional to the product of an integration photocarriers and the conversion gain, the reduction of junction capacitance must outweigh the loss of photocurrent arising from the substrate opening in order to benefit from employing this photodiode shape. Fig. C.3 shows an illustration of the reduction in junction capacitance with substrate opening. The plotted values were calculated based on reported junction capacitance parameters from a standard 0.35 µm and 0.18 µm CMOS technologies.

Fig. C.2 Illustration of photodiodes with one or more substrate openings.
A set of 20 \( \mu m \times 20 \mu m \) \( n^+\)-\( p_{epi} \) photodiodes were fabricated on a standard 0.35 \( \mu m \) CMOS process with various substrate openings (Fig. C.4). Fig. C.5 plots photocurrent versus the area of substrate opening. It can be seen that the reductions in photocurrent vary linearly with the area of the substrate opening relatively independent of the illumination wavelength. The linear reduction of photocurrent with the area of substrate opening suggests that the surface recombination occurring at the substrate opening is responsible for the reduction of photocurrent.
Fig. C.4  Micrographs taken from some of fabricated photodiodes. Each photodiode shape was implemented in a linear array to reduce the effects of mismatch.

Fig. C.6 plots the ratio between the photocurrent and the photodiode capacitance. Despite the reduction of photocurrent, a small net gain in the pixel’s signal output was observed for sufficiently large substrate openings. Fig. C.6 reveals that the reduction in junction capacitance does not overly compensate for the loss of photocurrent.

Another set of 20 µm x 20 µm n⁺-p_epi photodiodes with multiple 4 µm x 4 µm openings were also fabricated. Fig. C.7 plots the measured photocurrent from the photodiodes with multiple 4 µm x 4 µm openings along with the previous results obtained with single substrate opening.
Fig. C.5 Normalized photocurrent versus area of single square-shaped substrate opening on 20 µm x 20 µm n⁺-pₒᵦ photodiode.
Fig. C.6 Measured photocurrent divided by the photodiode junction capacitance (with side-wall junction considerations).
Fig. C.7 Normalized photocurrent versus the sum areas of multiple substrate openings. The photodiodes with multiple 4 µm x 4 µm openings display similar photocurrent reduction with the total area of substrate opening as in the case of single large opening.
Fig. C.8 Measured photocurrent divided by the photodiode junction capacitance (with side-wall junction considerations).

Another set of 20 µm x 20 µm n⁻-p_epi photodiodes were fabricated with multiple 1 µm x 1 µm openings in order to investigate whether the vertical side-wall junctions introduced by the openings can effectively enhance the photo-collection and thereby minimize the loss of photocurrent from the surface recombination. By applying 72 instances of 1 µm x 1 µm openings to each 20 µm x 20 µm photodiode, the sum length of side-walls increased from 80 µm to 368 µm. Fig. C.9 plots the ratio between the measured photocurrent and the resulting junction
capacitance as before. Despite the significant difference in the photodiode shape introduced by the small substrate openings, only a modest improvement in the final photocurrent to junction capacitance ratio was observed.

![Graph showing normalized photocurrent vs sum area of substrate openings](image)

**Fig. C.9** Normalized photocurrent versus sum area of substrate openings. The measurements from 20 µm x 20 µm $n^+$-$p_{eq}$ photodiode with 72 instances of 1 µm x 1 µm opening is shown above (enclosed in dotted lines) illustrating a modest improvement.

In summary, some modest improvements in conversion efficiency were demonstrated with single and multiple substrate openings in CMOS photodiodes. For processes employing silicide layers, the substrate openings can also enhance the photo-collection by allowing light to pass through the
opening. Whether or not these gains will increase as CMOS technology develops depend on a number of factors, including surface recombination, minority carrier diffusion length, and the employed photodiode size.
Appendix D

Simulation of Edge-Effect in Two-Dimensional Photodiode Array

TITLE Effects of high-low junction of epi-substrate + interface on the steady-state photoresponse

COMMENT + Updated parameters for 0.35um technology. + by Ji-Soo Lee + on 02/06/2001

COMMENT Variable declarations
ASSIGN NAME=EPITHCK N.VALUE=3.0
ASSIGN NAME=SUBTHCK N.VALUE=10.0
ASSIGN NAME=DOPSUB N.VALUE=2.0E15
ASSIGN NAME=DOPEPI N.VALUE=@DOPSUB/10
ASSIGN NAME=DOPNPLUS N.VALUE=1.0E20
ASSIGN NAME=DOPFPPLUS N.VALUE=1.0E20
ASSIGN NAME=JUNCDEF N.VALUE=0.10
ASSIGN NAME=V N.VALUE=1.0
ASSIGN NAME=XZCHAR N.VALUE=0.1
ASSIGN NAME=XGRDSZ N.VALUE=0.50
ASSIGN NAME=A N.VALUE=2.5

LOOP STEPS=1
ASSIGN NAME=YDEN N.VALUE=(0.4)

LOOP STEPS=4
ASSIGN NAME=BX + N.VALUE=(9,10,11,12) $+ N.VALUE=(0.1,0.17,0.5,1,1.5,2,2.5,3,4,4.5,5,5.5,6,7,8)

LOOP STEPS=4
ASSIGN NAME=BZ + N.VALUE=(0.2,1,2,3)
ASSIGN NAME=C N.VALUE=0.5

ASSIGN NAME=XMAX N.VALUE=4*@A+2*@BX
ASSIGN NAME=ZMAX N.VALUE=4*@A+2*@BZ
ASSIGN NAME=XC N.VALUE=@XMAX/2
ASSIGN NAME=ZC N.VALUE=@ZMAX/2

ASSIGN NAME=XR N.VALUE=@XC+2*@A+@BX
ASSIGN NAME=XT N.VALUE=@XC+2*@A+@BZ
ASSIGN NAME=XL N.VALUE=@XC-2*@A-@BX
ASSIGN NAME=ZL N.VALUE=@ZC-2*@A-@BZ

ASSIGN NAME=XR2 N.VALUE=@XR+2*@A+@BX
ASSIGN NAME=XT2 N.VALUE=@XT+2*@A+@BZ
ASSIGN NAME=XL2 N.VALUE=@XL-2*@A-@BX
ASSIGN NAME=ZL2 N.VALUE=@ZL-2*@A-@BZ

LOOP STEPS=1
ASSIGN NAME=SRV N.VALUE=(10)
COMMENT Specify VTH adjust doping and use concentration dependent mobility model CCSMOB to adjust mobility near Si-SiO2 interface

LOOP STEPS=1
ASSIGN NAME=VTHAD N.VALUE=1.4E17
$+ N.VALUE=(1.2E17,1.4E17,1.5E17,1.6E17,1.8E17,2.0E17)

COMMENT Create an initial simulation mesh
MESH REC
X.MESH X.MAX=@XMAX H1=@XGRDSZ
Z.MESH Z.MAX=@ZMAX H1=@XGRDSZ
Y.MESH N=1 L=-0.025
Y.MESH N=3 L=0.
Y.MESH DEPTH=@EPITHCK H1=0.02 H2=@YDEN
Y.MESH DEPTH=@SUBTHCK H1=@YDEN H2=5.00

COMMENT Region
REGION NUM=1 Y.MAX=@EPITHCK SILICON
REGION NUM=2 Y.MIN=@EPITHCK SILICON
REGION NUM=3 OXIDE IY.MAX=3

COMMENT Specify Electrodes
ELECTR NAME=ERC IY.MAX=3
+ X.MIN=@XR-@C X.MAX=@XR+@C
+ Z.MIN=@ZC-@C Z.MAX=@ZC+@C
ELECTR NAME=ECC IY.MAX=3
+ X.MIN=@XC-@C X.MAX=@XC+@C
+ Z.MIN=@ZC-@C Z.MAX=@ZC+@C
ELECTR NAME=ELC IY.MAX=3
+ X.MIN=@XL-@C X.MAX=@XL+@C
+ Z.MIN=@ZC-@C Z.MAX=@ZC+@C
ELECTR NAME=ECT IY.MAX=3
+ X.MIN=@XC-@C X.MAX=@XC+@C
+ Z.MIN=@ZT-@C Z.MAX=@ZT+@C
ELECTR NAME=ECS IY.MAX=3
+ X.MIN=@XC-@C X.MAX=@XC+@C
+ Z.MIN=@ZB-@C Z.MAX=@ZB+@C
ELECTR NAME=ERT IY.MAX=3
+ X.MIN=@XR-@C X.MAX=@XR+@C
+ Z.MIN=@ZT-@C Z.MAX=@ZT+@C
ELECTR NAME=ERB IY.MAX=3
+ X.MIN=@XR-@C X.MAX=@XR+@C
+ Z.MIN=@ZB-@C Z.MAX=@ZB+@C
ELECTR NAME=ELT IY.MAX=3
+ X.MIN=@XL-@C X.MAX=@XL+@C
+ Z.MIN=@ZT-@C Z.MAX=@ZT+@C
ELECTR NAME=ELB IY.MAX=3
+ X.MIN=@XL-@C X.MAX=@XL+@C
+ Z.MIN=@ZB-@C Z.MAX=@ZB+@C

ELECTR NAME=Substrate BOTTOM

COMMENT Specify impurity profiles
PROFILE P-TYPE N.PEAK=@DOPEPI UNIF OUT.FILE=ds
PROFILE P-TYPE N.PEAK=@VTHAD Y.CHAR=.25
PROFILE P-TYPE N.PEAK=@DOPSUB UNIF
+ Y.MIN=@EPITCK

PROFILE N-TYPE N.PEAK=@DOPNPLUS
+ Y.MIN=0 Y.JUNC=@JUNCDEP XY.RAT=0.75 ZY.RAT=0.75
+ X.MIN=@XC-@A X.MAX=@XC+@A
+ Z.MIN=@ZC-@A Z.MAX=@ZC+@A

PROFILE N-TYPE N.PEAK=@DOPNPLUS
+ Y.MIN=0 Y.JUNC=@JUNCDEP XY.RAT=0.75 ZY.RAT=0.75
+ X.MIN=@XR-@A X.MAX=@XR+@A
+ Z.MIN=@ZT-@A Z.MAX=@ZT+@A

COMMENT Slightly alter some of the material parameters

MATERIAL REGION=1 TAUN0=2.35E-5
MATERIAL REGION=2 TAUN0=4.11E-9
MOBILITY SILICON MUN0=1.5E3

COMMENT SiO2/Si Interface
INTERFAC S.N=@SRV

COMMENT Refine the mesh with doping regrids
REGRID DOPING LOG RAT=1 SMOOTH=1 IN.FILE=ds
+ OUT.FILE="mesh"

COMMENT Display the simulation mesh
$PLOT.2D TITLE="Mesh" FILL Y.MAX=@SUBTHCK
$PLOT.1D TITLE="Doping Concentration"
$+ DOP LOG X.START=0.25 X.END=0.25 Y.START=0 Y.END=@SUBTHCK
$PLOT.3D BOX TITLE="Simulation Mesh"
$ BOXカメラ=(−5,−5,−5)
$PLOT.2D GRID TITLE="Simulation Mesh"
$ GRID SCALE Y.PLANE=0.05
$PLOT.2D GRID TITLE="Simulation Mesh"
$+ FILL\ SCALE\ X.\ PLANE=0.05$

$PLOT.2D\ GRID\ TITLE="Simulation\ Mesh"$

$+ FILL\ SCALE\ Z.\ PLANE=0.05$

COMMENT\ Specify\ physical\ models\ to\ use
MODELS\ SRH\ AUGER\ FLDMOB\ BGN\ CCSMOB

COMMENT\ Calculate\ a\ steady\ state\ solution\ with\ reverse\ bias
+\ of\ 1.0\ Volts.\ Perform\ a\ zero\ carrier\ solution\ to\ use
+\ as\ the\ initial\ guess\ for\ the\ two\ carrier\ solution.
SYMB\ GUMMEL\ CARRIERS=0
METHOD\ DAMPED\ ICCG
SOLVE\ V(\ Substrate)=0
  +\ V(ELT)=@V\ V(BCT)=@V\ V(BRT)=@V
  +\ V(ELC)=@V\ V(BCC)=@V\ V(BRC)=@V
  +\ V(ELB)=@V\ V(BCB)=@V\ V(BRB)=@V

COMMENT\ Assign\ names\ and\ values\ for\ the\ characteristic
+\ absorption\ distance\ and\ the\ photon\ flux.
ASSIGN\ NAME=W\ N.VALUE=540
ASSIGN\ NAME=Y.CHAR\ N.VALUE=0.910
ASSIGN\ NAME=FLUX\ N.VALUE=6.690E13

COMMENT\ Illuminate
PHOTOGEN\ A3=1E4*@FLUX/@Y.CHAR\ A4=-1/@Y.CHAR

COMMENT\ Open\ a\ log\ file\ for\ storing\ the\ terminal\ data
LOG\ OUT.FILE="pd2d"@VTHAD"z"@BZ"x"@BX".txt"

EXTRACT\ NAME=IPH\ EXPR="@I(ECC)"

COMMENT\ Switch\ to\ Newton\ and\ two\ carriers\ and\ solve\ for\ the
+\ steady\ state\ reverse\ bias\ solution.
SYMBOL\ NEWTON\ CARRIERS=2
SOLVE\ V(\ Substrate)=0
  +\ V(ELT)=@V\ V(BCT)=@V\ V(BRT)=@V
  +\ V(ELC)=@V\ V(BCC)=@V\ V(BRC)=@V
  +\ V(ELB)=@V\ V(BCB)=@V\ V(BRB)=@V

COMMENT\ Plot\ the\ excess\ minority\ carrier\ density
$PLOT.1D\ ELECTRON\ LOG$
$+\ X.START=0.25\ X.END=0.25$
$+\ Y.START=@EPITHCK\ Y.END=@EPITHCK+5.0$
$+\ OUT.FILE="exelec"@W".txt"

$PLOT.1D\ J.ELEC$
$+\ X.START=0.25\ X.END=0.25$
$+\ Y.START=@EPITHCK\ Y.END=@EPITHCK+0.1$
$+\ OUT.FILE="subcurr"@W".txt"

$PLOT.1D\ J.ELEC$
$+\ X.START=@PDWID2+@PDSEP+@PDWID/2$
$+\ X.END=@PDWID2+@PDSEP+@PDWID/2$
$+\ Y.START=@JUNCDEP+0.1\ Y.END=@JUNCDEP+0.2$
$+\ OUT.FILE="totcurr"@W".txt"

$PLOT.1D\ N.MOBILITY$
$+\ X.START=@PDWID2+@PDSEP/2$
$+\ X.END=@PDWID2+@PDSEP/2$
$+\ Y.START=0\ Y.END=0.5$
$+\ OUT.FILE="surmob"@VTHAI".txt"
$PLOT.1D  ELECTRON LOG
$+  X.START=@PDWID2+@PDSEP/2
$+  X.END=@PDWID2+@PDSEP/2
$+  Y.START=0  Y.END=7.0
$+  OUT.FILE="surele"@VTHAI".txt"
L.END
L.END
L.END
L.END
L.END
References


### Glossary

- $\alpha$: absorption coefficient
- $\eta$: quantum efficiency
- $\lambda$: wavelength of incident light
- $\theta$: angle of transmission with respect to normal
- $\Phi_s$: flux of diffused excess minority carriers from substrate injected into epitaxial layer
- $c$: speed of light
- $d$: distance between the bottom edge of the photodiode junction and Si-SiO$_2$ interface
- $F_0$: incident photon flux
- $G_o$: peak photo-generation rate at Si-SiO$_2$ interface
- $h$: Plank’s constant
- $J_{ph}$: photocurrent density
- $L_e$: excess minority carrier diffusion length of epitaxial layer
- $L_{op}$: optical absorption depth
- $L_s$: excess minority carrier diffusion length of substrate bulk
- $n_{air}$: index of refraction for air
- $n_{Si}$: index of refraction for silicon crystal
- $n_{SiO2}$: index of refraction for amorphous silicon dioxide
- $N_e$: uniform doping concentration of epitaxial layer
- $N_s$: uniform doping concentration of substrate bulk
- $\hat{n}_e$: excess minority carrier concentration in epitaxial layer
\( \hat{n}_s \) excess minority carrier concentration in substrate bulk

\( p \) optical path length difference

\( P_{in} \) incident power density

\( R \) reflectance

\( s \) distance between linear photodiode array

\( t \) epitaxial layer thickness

\( t_{ox} \) thickness of the oxide layer