

How to fix a double-black violation

$z \leftarrow$ child of deleted node (which is double-black)

loop

invariants: only possible violation of tree

properties is that z is double-black

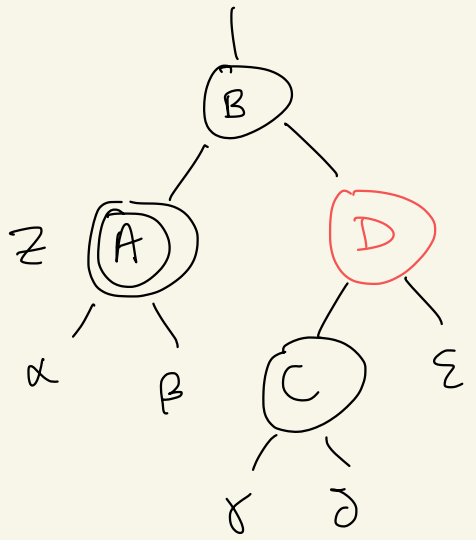
exit when $z = \text{root}$ or z is not extra black

do one of the local fixes on next slides

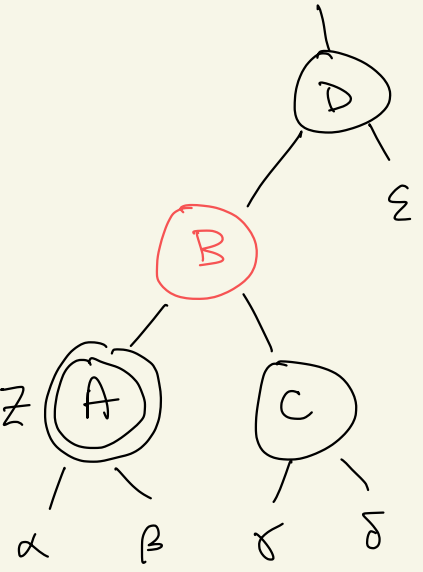
end loop

$z.\text{colour} \leftarrow \text{black}$

Case 1: z's sibling is red

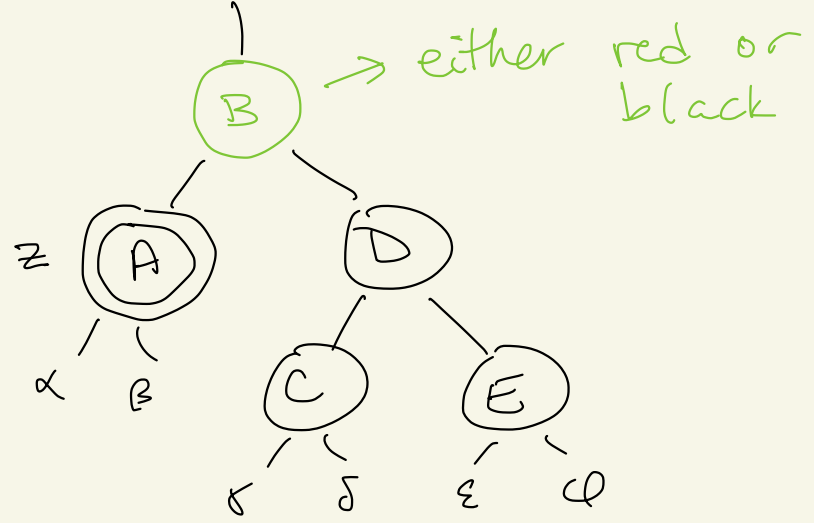


⇓ rotate & recolour

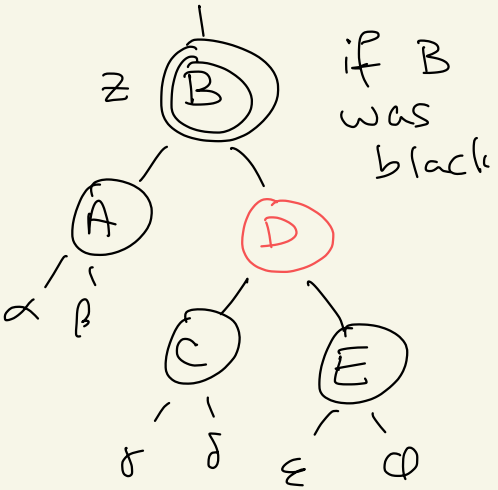


Now reduced to one of other cases

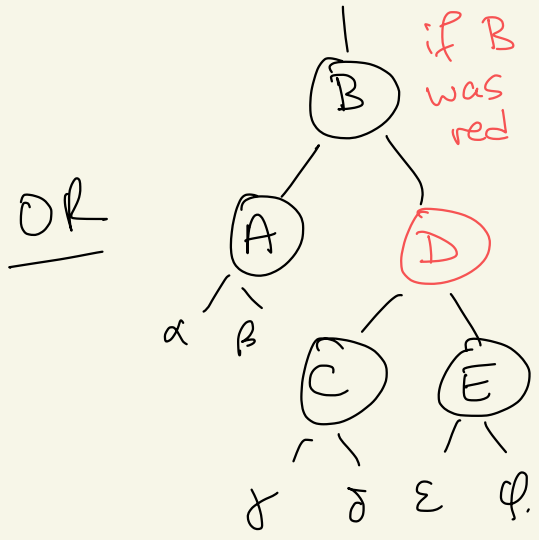
Case 2: z's sibling and both nephews black



⇓ recolour



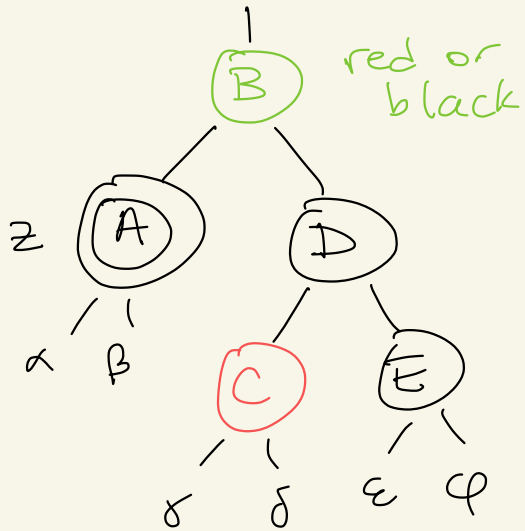
Violation z moves up tree



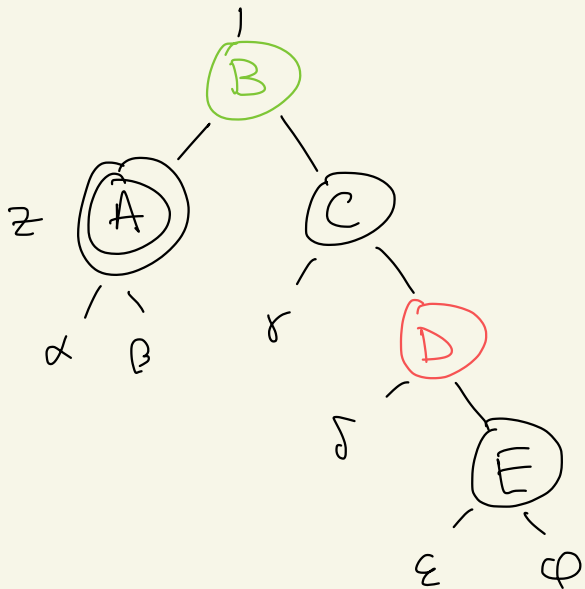
DONE!

Case 3 z's sibling black

z's near nephew red
z's far nephew black



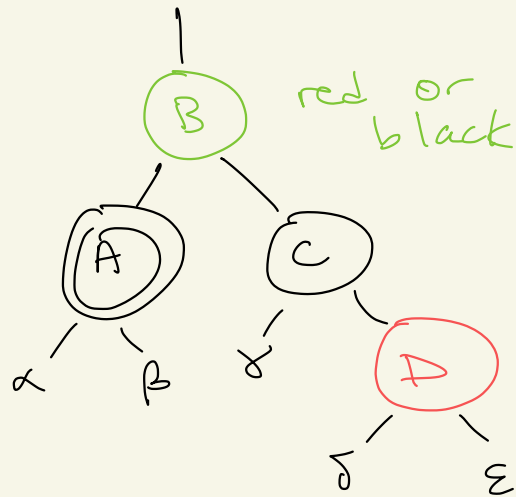
Rotated and recoloured



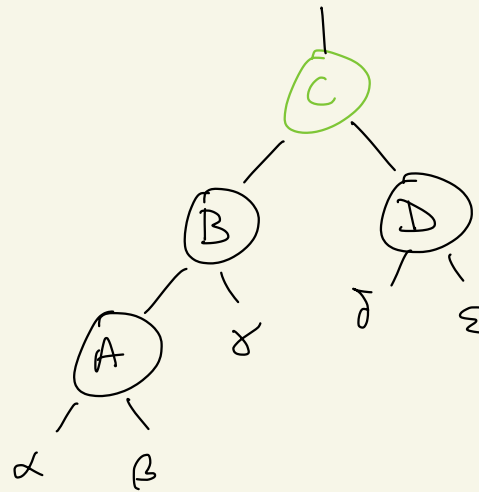
now reduced to Case 4

Case 4

z's sibling black
z's far nephew is red



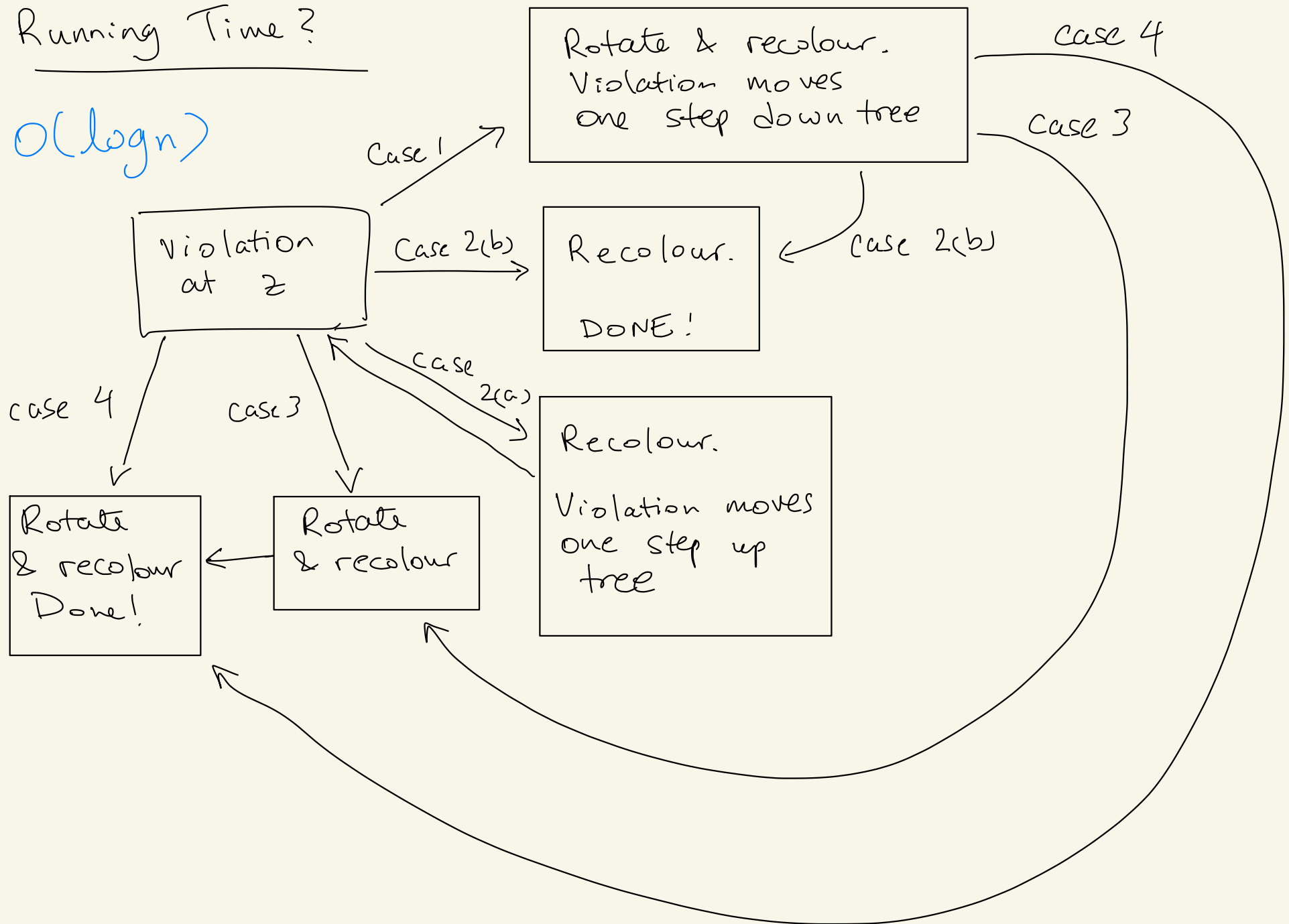
Rotated and recoloured



DONE!

Running Time?

$O(\log n)$



How many rotations?

At most 3.