# Homework Assignment #4
## Due: February 14, 2019 at 11:30 a.m.

**1.** Consider a data structure for disjoint sets that stores each set as an up-tree. Each node has a pointer to its parent.

Suppose we do *not* keep rank or size information. When two roots are linked by a UNION operation, we choose one of the two roots arbitrarily to make it the child of the other.

Instead of path compression (which requires two passes along the path from a node to the root), we make a single pass and update the parent pointer of each node to point to its former grandparent. This is sometimes called path splitting. For example, if there is a path $v_1 \to v_2 \to v_3 \to \cdots \to v_{2k}$, where $v_k$ is a root node, then a FIND$(v_1)$ will modify the parent pointers of these nodes so that after the FIND, there are two paths $v_2 \to v_4 \to \cdots \to v_{2k}$ and $v_1 \to v_3 \to \cdots \to v_{2k-1} \to v_{2k}$.

Now, suppose the data structure begins with $n$ nodes already linked together into trees in some way. Then, we do a sequence of $m$ FIND operations.

**(a)** Show that the total time for the sequence of FIND operations is $O(n \log n + m)$

**(b)** Show that for any $n$, there is a starting configuration of the data structure with $n$ nodes and a sequence of $n$ FIND operations that takes $\Omega(n \log n)$ time in total.

**2.** Once upon a time, Ronald Drumph was the president of a large country for many years. The first thing he did after becoming president was cut taxes for the richest people in his country. The resulting lack of funds required shutting down the government entirely for some periods of time.

As a result, the infrastructure in his country fell apart. In particular, some national highways fell into disrepair and became unusable. It got so bad that it eventually became impossible to travel between certain pairs of cities in the country. Mr. Drumph didn't much care, because he had a private plane that he could use to fly around the country (and there were some parts of the country that he didn't want to visit anyway).

We can model the highway network at the beginning of Mr. Drumph's term in office as a graph with $n$ nodes and $m$ edges, where each node represents a city and an edge connecting two nodes represents a highway between the two corresponding cities. Historians have compiled a list $(e_1, d_1), (e_2, d_2), \ldots, (e_k, d_k)$ indicating that edge $e_i$ in this graph became unusable on day $d_i$ of Mr. Drumph's term in office.

The historians would now like to figure out, for each day during Mr. Drumph's term, the size of the largest set of cities that is still connected by a road network (so that people can travel by road between any pair of cities in that set).

**(a)** Write an algorithm to solve this problem efficiently and explain why it is correct. Give detailed descriptions of how any data structures you use in your algorithm are implemented. The output should be $k+1$ lines of the form: from day $x$ to day $y$, the largest set of connected cities had size $z$.

**(b)** Give a good upper bound on the worst-case running time of your algorithm in terms of $k, m$ and/or $n$ using big-O notation.