Homework Assignment #3
Due: January 29, 2019 at 11:30 a.m.

1. Instead of using UNION to perform an INSERT into a binomial heap H, we can write code that directly does the INSERT. Note that we keep the list of roots of H sorted by degree.

```
1  INSERT(H, item)
2      let x be a pointer to a new node x with NIL in its parent, child and sibling pointers
3      x.key ← item
4      x.degree ← 0
5      loop
6          let y be a pointer to the first root in H's list of roots
7          exit when y = NIL or x.degree ≠ y.degree
8          remove the node that y points to from H's list of roots
9          if x.key > y.key then swap the pointers x and y
10         make node y the first child of node x
11     end loop
12     add node x to the beginning of H's list of roots
13 end INSERT
```

Use the potential method to show that the total time required to do a sequence of m INSERTS on an initially empty binomial heap is $O(m)$.

2. In class, we showed that a sequence of m INCREMENTS to a binary counter takes $O(m)$ bit flips in total. Professor Mickenfett observes that a symmetric argument can be used to show that if the binary counter initially has the value m, then a sequence of m decrements (taking it all the way down to value 0) also does $O(m)$ bit flips in total. After seeing your answer to part (a), she figures, by analogous reasoning, that a sequence of m EXTRACTMIN operations on a binomial heap that initially contains m items should also be doable in $O(m)$ steps.

(a) Show that this claim is incorrect for the standard implementation of EXTRACTMIN. (Recall that the standard implementation of EXTRACTMIN removes the tree with the minimum root r from the binomial heap H's list of roots, forms a new binomial heap H' from the list of r's children and then does a UNION of H and H'.)

(b) Prof. Mickenfett then writes a *new and improved* implementation of EXTRACTMIN for binomial heaps. Her implementation of EXTRACTMIN is fully general: it makes no assumptions about what the priority values are (except that they come from an ordered set, so that talking about the minimum priority makes sense). She claims that, starting from an arbitrary binomial heap containing m elements, a sequence of m EXTRACTMINS takes $O(m)$ steps in total. Explain why Prof. Mickenfett is mistaken.