EECS-1019C: ASSIGNMENT #9

Out of N points.

Section 6.1 [12pt]

8. [2pt] How many different three-letter initials with none of the letters repeated can people have?

$\mathbf{P}(26, 3) = 26 \cdot 25 \cdot 24 = 15600$

12. [2pt] How many bit strings are there of length six or less, not counting the empty string?

$2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 = 2^7 - 2 = 126$

32. [6pt] How many strings of eight uppercase English letters are there

b. [2pt] if no letter can be repeated?

$\mathbf{P}(26, 8) = 26!/(26 - 8 + 1)! = 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 62990928000$

d. [2pt] that start with X, if no letter can be repeated?

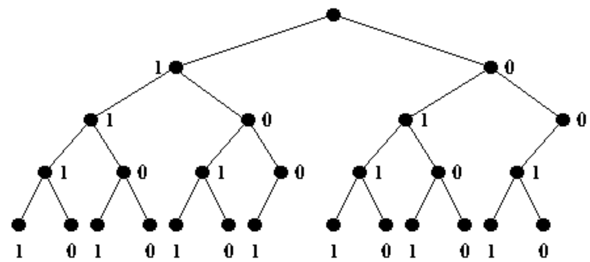
$\mathbf{P}(25, 7) = 25!/(25 - 7 + 1)! = 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 = 2422728000$

f. [2pt] that start with the letters BO (in that order), if letters can be repeated?

$26^6 = 308915776$

64. [2pt] Use a tree diagram to find the number of bit strings of length four with no three consecutive 0s.

There are 2 bit-strings of length 4 that commence with “000”, 2 end with “000”; “0000” is double counted, so three in all that have “000”. There are $2^4 = 16$ *bit-strings in total, so* $16 - 3 = 13$ *that qualify.*



Section 6.3 [8pt]

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4. [4pt] *(Missed on hardcopy posting. Comped.)* Let $S = \{1, 2, 3, 4, 5\}$.

a. [2pt] List all the 3-permutations of S.

(1, 2, 3), (2, 3, 1), (3, 1, 2), (1, 3, 2), (3, 2, 1), (2, 1, 3),
(1, 2, 4), (2, 4, 1), (4, 1, 2), (1, 4, 2), (4, 2, 1), (2, 1, 4),
(1, 2, 5), (2, 5, 1), (5, 1, 2), (1, 5, 2), (5, 2, 1), (2, 1, 5),
(1, 3, 4), (3, 4, 1), (4, 1, 3), (1, 4, 3), (4, 3, 1), (3, 1, 4),
(1, 3, 5), (3, 5, 1), (5, 1, 3), (1, 5, 3), (5, 3, 1), (3, 1, 5),
(1, 4, 5), (4, 5, 1), (5, 1, 4), (1, 5, 4), (5, 4, 1), (4, 1, 5),
(2, 3, 4), (3, 4, 2), (4, 2, 3), (2, 4, 3), (4, 3, 2), (3, 2, 4),
(2, 3, 5), (3, 5, 2), (5, 2, 3), (2, 5, 3), (5, 3, 2), (3, 2, 5),
(2, 4, 5), (4, 5, 2), (5, 2, 4), (2, 5, 4), (5, 4, 2), (4, 2, 5),
(3, 4, 5), (4, 5, 3), (5, 3, 4), (3, 5, 4), (5, 4, 3), (4, 3, 5)

b. [2pt] List all the 3-combinations of S.

$\{1, 2, 3\}, \{1, 2, 4\}, \{1, 2, 5\}, \{1, 3, 4\}, \{1, 3, 5\},$
$\{1, 4, 5\}, \{2, 3, 4\}, \{2, 3, 5\}, \{2, 4, 5\}, \{3, 4, 5\}$

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14. [2pt] In how many ways can a set of two positive integers less than 100 be chosen?

$$\binom{99}{2} = 4851$$

26. [6pt] Thirteen people on a softball team show up for a game.

a. [2pt] How many ways are there to choose 10 players to take the field?

$$\binom{13}{10} = 286$$

b. [2pt] How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?

$$\mathbf{P}(13, 10) = 13 \cdot 12 \cdot \ldots \cdot 4 = 1037836800$$

c. [2pt] Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

Number of ways to choose any team minus number of ways to choose the team with just men:

$$\binom{13}{10} - \binom{10}{10} = 285$$

Or a more long-winded way, but correct:

$$\binom{3}{1}\binom{10}{9} + \binom{3}{2}\binom{10}{8} + \binom{3}{3}\binom{10}{7} = 3 \cdot 10 + 3 \cdot 45 + 1 \cdot 105 = 30 + 135 + 120 = 285$$

Section 6.4 [6pt]

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4. [2pt] Find the coefficient of x^5y^8 in $(x+y)^13$.

$\binom{13}{5} = 1287$

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6. [2pt] What is the coefficient of x^7 in $(1+x)^{11}$?

$\binom{11}{7} = 330$

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14. [2pt] Show that if n is a positive integer, then $1 = \binom{n}{0} < \binom{n}{1} < \ldots < \binom{n}{\lfloor n/2 \rfloor} = \binom{n}{\lceil n/2 \rceil} > \ldots > \binom{n}{n-1} > \binom{n}{n} = 1$.

We know that $\binom{n}{0} = \binom{n}{n} = 1$. *If* n *is even, then* $\binom{n}{\lfloor n/2 \rfloor} = \binom{n}{\lceil n/2 \rceil}$ *as* $\lfloor n/2 \rfloor = \lceil n/2 \rceil$. *If* n *is odd, then* $\binom{n}{\lfloor n/2 \rfloor} = \frac{n!}{\lfloor n/2 \rfloor ! \lceil n/2 \rceil !} = \binom{n}{\lceil n/2 \rceil}$.
Now we simply need to show that $\binom{n}{i-1} < \binom{n}{i}$ *for* $1 < i \leq \lfloor n/2 \rfloor$. $\binom{n}{i-1} = \frac{n!}{(i-1)!(n-i+1)!}$ *and* $\binom{n}{i} = \frac{n!}{i!(n-i)!}$. *Thus,* $\binom{n}{i} = ((n-i+1)/i)\binom{n}{i-1}$. $(n-i+1)/i = (n-(i-1))/i > 1$ *as* $i \leq \lfloor n/2 \rfloor$. *Thus,* $\binom{n}{i} > \binom{n}{i-1}$.
By symmetry of the binomial, it immediately follows that $\binom{n}{i} > \binom{n}{i+1}$ *for* $\lceil n/2 \rceil \geq i < n$.