EECS-1019C: ASSIGNMENT #6

Out of 25 points.

Section 3.1 [10pt]

4. [5pt] Describe an algorithm that takes as input a list of n integers and produces as output the largest difference obtained by subtracting an integer in the list from the one following it.

```
1  largestDiff(a_1, ..., a_n)
2      diff := a_2 - a_1
3      for i := 2, ..., n - 1
4          if (a_{i+1} - a_i > diff)
5              diff := a_{i+1} - a_i
6      return diff
```

12. [5pt] Describe an algorithm that uses only assignment statements that replaces the triple (x, y, z) with (y, z, x). What is the minimum number of assignment statements needed?

```
1  t := z
2  x := y
3  y := z
4  z := t
```

where t is a temporary variable.

Section 3.2 [15pt]

18. [5pt] Let k be a positive integer. Show that $1^k + 2^k + \ldots + n^k$ is $\mathcal{O}(n^{k+1})$.

$1^k + 2^k + \ldots + (n-1)^k + n^k < n^k + n^k + \ldots + n^k + n^k = n^{k+1}$

30. [10pt] Show that each of these pairs of functions are of the same order.

a. [2pt] $3x + 7$, x

$x \leq 3x + 7 \leq 4x$ *for all* $x > 7$.

b. [2pt] $2x^2 + x - 7$, x^2

$x^2 < 2x^2 + x - 7$ *for* $x \leq 7$. $2x^2 + x - 7 \leq 3x^2$ *for* $x \leq 1$.

c. [2pt] $\lfloor x + 1/2 \rfloor$, x

$\lfloor x + 1/2 \rfloor \leq 2x$ *for* $x > 2$. $x \leq 2\lfloor x + 1/2 \rfloor$ *for* $x > 2$.

d. [2pt] $\log(x^2 + 1)$, $\log_2(x)$

Note "$\log$*" is the same as "*$\log_2$*".*
$\log_2(x^2 + 1) \leq \log_2(2x^2) = 1 + 2\log_2(x) \leq 3\log_2(x)$ *for* $x > 2$. *Since* $x < x^2 + 1$ *for all* $x > 1$, *it follows that* $\log_2(x) < \log_2(x^2 + 1)$.

e. [2pt] $\log_{10} x$, $\log_2 x$

Follows from $\log_{10} x = C \log_2 x$ *where* $C = 1/\log_2 10$.