EECS-1019c: Assignment #3

Out of 40 points.

Section 1.6 [12pt]

8. [4pt] What rules of inference are used in this argument? "No man is an island. Manhattan is an island. Therefore, Manhattan is not a man."

> *Universal Generalization:* $\forall x(Man(x) \rightarrow \neg Island(x))$. *Instantiation with Manhattan. Modus tollens with the fact* $island(Manhattan)$.

14. [4pt] For each of these arguments, explain which rules of inference are used for each step.

a. "Linda, a student in this class, owns a red convertible. Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket."

> $c(x)$: *x is in this class.*
> $r(x)$: *x owns a red convertible.*
> $t(x)$: *x has gotten a speeding ticket.*

1. $\forall x(r(x) \rightarrow t(x))$	*Hypothesis*
2. $r(Linda) \rightarrow t(Linda)$	*Universal instantiation using (1)*
3. $r(Linda)$	*Hypothesis*
4. $t(Linda)$	*Modus ponens using (2) and (3)*
5. $c(Linda)$	*Hypothesis*
6. $r(Linda) \wedge c(Linda)$	*Conjunction using (4) and (5)*
7. $\exists x(c(x) \wedge t(x))$	*Existential generalization using (6)*

b. "Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course in discrete mathematics can take a course in algorithms. Therefore, all five roommates can take a course in algorithms next year."

> $r(x)$: *x is one of the five roommates.*
> $d(x)$: *x has taken a course in discrete math.*
> $a(x)$: *x can take a course in algorithms.*

1. $\forall x(r(x) \rightarrow d(x))$	*Hypothesis*
2. $r(c) \rightarrow d(c)$	*Universal instantiation using (1)*
3. $\forall x(d(x) \rightarrow a(x))$	*Hypothesis*
4. $d(c) \rightarrow a(c)$	*Universal instantiation using (3)*
5. $r(c) \rightarrow a(c)$	*Hypothetical syllogism using (2) and (4)*
6. $\forall x(r(x) \rightarrow a(x))$	*Universal generalization using (5)*

c. "All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners. Therefore, there is a wonderful movie about coal miners."

$s(x)$: x is movie by Sayles.
$c(x)$: x is about coal miners.
$w(x)$: x is wonderful.

1. $\exists x(s(x) \wedge c(x))$	*Hypothesis*
2. $s(m) \wedge c(m)$	*Existential instantiation using (1)*
3. $s(m)$	*Simplification using (2)*
4. $\forall x(s(x) \rightarrow w(x))$	*Hypothesis*
5. $s(m) \rightarrow w(m)$	*Universal instantiation using (4)*
6. $w(m)$	*Modus ponens using (3) and (5)*
7. $c(m)$	*Simplification using (2)*
8. $w(m) \wedge c(m)$	*Conjunction using (6) and (7)*
9. $\exists x(w(x) \wedge c(x))$	*Existential generalization using (8)*

d. "There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre."

$c(x)$: x is in the class.
$f(x)$: x has been to France.
$l(x)$: x has visited the Louvre.

1. $\exists x(c(x) \wedge f(x))$	*Hypothesis*
2. $c(s) \wedge f(s)$	*Existential instantiation using (1)*
3. $f(s)$	*Simplification using (2)*
4. $c(s)$	*Simplification using (2)*
5. $\exists x(f(x) \rightarrow l(x))$	*Hypothesis*
6. $\exists f(s) \rightarrow l(s)$	*Universal instantiation using (5)*
7. $l(s)$	*Modus ponens using (3) and (6)*
8. $c(s) \wedge l(s)$	*Conjunction using (4) and (7)*
9. $\exists x(c(x) \wedge l(x))$	*Existential generalization using (8)*

24. [4pt] Identify the error or errors in this argument that supposedly shows that if $\forall x(P(x) \vee Q(x))$ is true then $\forall x(P(x)) \vee (\forall x(Q(x))$ is true.

1. $\forall x(P(x) \vee Q(x))$	Premise
2. $P(c) \vee Q(c)$	Universal Instantiation from (1)
3. $P(c)$	Simplification from (2)
4. $\forall x(P(x))$	Universal Generalization from (3)
5. $Q(c)$	Simplification from (2)
6. $\forall x(Q(x))$	Universal Generalization from (5)
7. $\forall x(P(x)) \vee (\forall x(Q(x))$	Conjunction from (4) and (5)

Steps 3 and 5 are incorrect. Simplification applies to conjunctions, not disjunctions.

Section 1.7 [20pt]

6. [4pt] Use a direct proof to show that the product of two odd numbers is odd.

An odd number has the form $2n+1$ where n is an integer. Consider $2x+1$ and $2y+1$, two odd numbers, for some integers x and y. $(2x+1)(2y+1) = 4xy+2x+2y+1$. $4xy+2x+2y = 2(2xy+x+y)$, so is even. Thus, $4xy+2x+2y+1$ is an odd number.

8. [4pt] Prove that if n is a perfect square, then $n+2$ is not a perfect square.

Let $n = m^2$. If $m = 0$, then $n+2 = 2$. 2 is not a perfect square. Thus, assume $m \geq 1$. The smallest perfect square greater than n is $(m+1)^2$. $(m+1)^2 = m^2+2m+1 > n+2\cdot1+1 = n+3 > n+2$.

16. [4pt] Prove that if m and n are integers and mn is even, then m is even or n is even.

Proof by contraposition. Assume it is not the case that m is even or n is even. Thus, both are odd. By the proof for #6 above, mn is odd.

24. [4pt] Show that at least three of any 25 days chosen must fall in the same month of the year.

Proof by contradiction. If there were at most two days falling in the same month, then we could have at most $2\cdot12 = 24$ days, since there are twelve months. As we have chosen 25 days, at least three must fall in the same month.

26. [4pt] Prove that if n is a positive integer, then n is even if and only if $7n+4$ is even.

We need to prove both directions.
We show that, if n is even, then $7n+4$ is even. As n is even, $n = 2k$ for some integer k. $7n+4 = 7\cdot2k+4 = 14k+4 = 2(7k+2)$. Thus, $7n+4$ is even.
We show that, if $7n+4$ is even, then n is even. Assume n is odd. Thus, $n = 2k+1$ for some integer k. $7n+4 = 7(2k+1)+4 = 14k+7+4 = 2(7k+5)+1$. $2(7k+5)$ is even, so $2(7k+5)+1$ is odd. Contradiction. Thus, n is even.

Section 1.8 [8pt]

4. [4pt] Use a proof by cases to show that $\min(a, \min(b, c)) = \min(\min(a, b), c)$ whenever a, b, and c are real numbers.

There are three cases:

1. *that a is smaller than or equal to b and to c;*
2. *that b is smaller than or equal to a and to c; and*
3. *that c is smaller than or equal to a and to b.*

C_1 *We have two cases:*

- A. *that b is smaller than or equal to c; and*
- B. *that c is smaller than or equal to b.*

C_A $min(a, min(b, c)) = min(a, b) = a.$
$min(min(a, b), c) = min(a, c) = a.$

C_B $min(a, min(b, c)) = min(a, c) = a.$
$min(min(a, b), c) = min(a, c) = a.$

C_2 *We have two cases:*

- A. *that a is smaller than or equal to c; and*
- B. *that c is smaller than or equal to a.*

C_A $min(a, min(b, c)) = min(a, b) = b.$
$min(min(a, b), c) = min(b, c) = b.$

C_B $min(a, min(b, c)) = min(a, b) = b.$
$min(min(a, b), c) = min(b, c) = b.$

C_3 *We have two cases:*

- A. *that a is smaller than or equal to b; and*
- B. *that b is smaller than or equal to a.*

C_A $min(a, min(b, c)) = min(a, c) = c.$
$min(min(a, b), c) = min(a, c) = c.$

C_B $min(a, min(b, c)) = min(a, c) = c.$
$min(min(a, b), c) = min(b, c) = c.$

14. [4pt] Prove or disprove that if a and b are rational numbers, then a^b is also rational.

We prove this is not true by counterexample. Consider $a = 2$ and $b = \frac{1}{2}$. $a^{\frac{1}{2}} = \sqrt{2}$. It is known that $\sqrt{2}$ is irrational.