EECS-1019C: ASSIGNMENT #2

Out of 30 points.

Section 1.4 [18pt]

8. [8pt] Translate these statements into English, where R(x) is "x is a rabbit" and H(x) is "x hops" and the domain consists of all animals.

a. [2pt] $\forall x(\mathbf{R}(x) \rightarrow \mathbf{H}(x))$

All rabbits hop.

b. [2pt] $\forall x(\mathbf{R}(x) \wedge \mathbf{H}(x))$

All animals are rabbits that hop.

c. [2pt] $\exists x(\mathbf{R}(x) \rightarrow \mathbf{H}(x))$

So, there is an animal; if there is only a single animal in the whole world which is a rabbit, it hops.

d. [2pt] $\exists x(\mathbf{R}(x) \wedge \mathbf{H}(x))$

There is a rabbit that hops.

20. [10pt] Suppose that the domain of the propositional function P(x) consists of −5, −3, −1, 1, 3, and 5. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

a. [2pt] $\exists x \mathbf{P}(x)$

$\mathbf{P}(-5) \vee \mathbf{P}(-3) \vee \mathbf{P}(-1) \vee \mathbf{P}(1) \vee \mathbf{P}(3) \vee \mathbf{P}(5)$

b. [2pt] $\forall x \mathbf{P}(x)$

$\mathbf{P}(-5) \wedge \mathbf{P}(-3) \wedge \mathbf{P}(-1) \wedge \mathbf{P}(1) \wedge \mathbf{P}(3) \wedge \mathbf{P}(5)$

c. [2pt] $\forall x((x \neq 1) \rightarrow \mathbf{P}(x))$

$\mathbf{P}(-5) \wedge \mathbf{P}(-3) \wedge \mathbf{P}(-1) \wedge \mathbf{P}(3) \wedge \mathbf{P}(5)$

d. [2pt] $\exists x((x \geq 0) \wedge \mathbf{P}(x))$

$\mathbf{P}(1) \vee \mathbf{P}(3) \vee \mathbf{P}(5)$

e. [2pt] $\exists x(\neg \mathbf{P}) \wedge \forall x((x < 0) \rightarrow \mathbf{P}(x))$

$(\mathbf{P}(1) \vee \mathbf{P}(3) \vee \mathbf{P}(5)) \wedge (\neg \mathbf{P}(-5) \wedge \neg \mathbf{P}(-3) \wedge \neg \mathbf{P}(-1))$

24. [10pt] Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

> *Let* $\mathsf{C}(x)$ *mean* x *is a student in your class.*

a. [2pt] Everyone in your class has a cellular phone.

> *Let* $\mathsf{P}(x)$ *mean* x *has a cellular phone.*
> $\forall x \mathsf{P}(x)$
> $\forall x(\mathsf{C}(x) \to \mathsf{P}(x))$

b. [2pt] Somebody in your class has seen a foreign movie.

> *Let* $\mathsf{F}(x)$ *mean* x *has seen a foreign movie.*
> $\exists x \mathsf{F}(x)$
> $\exists x(\mathsf{C}(x) \land \mathsf{F}(x))$

c. [2pt] There is a person in your class who cannot swim.

> *Let* $\mathsf{S}(x)$ *mean* x *can swim.*
> $\exists x(\neg \mathsf{S}(x))$
> $\exists x(\mathsf{C}(x) \land \neg \mathsf{S}(x))$

d. [2pt] All students in your class can solve quadratic equations.

> *Let* $\mathsf{Q}(x)$ *mean* x *can solve quadratic equations.*
> $\forall x \mathsf{Q}(x)$
> $\forall x(\mathsf{C}(x) \to \mathsf{Q}(x))$

e. [2pt] Some student in your class does not want to be rich.

> *Let* $\mathsf{R}(x)$ *mean* x *does not want to be rich.*
> $\exists x(\neg \mathsf{R}(x))$
> $\exists x(\mathsf{C}(x) \land \neg \mathsf{R}(x))$

Section 1.5 [12pt]

6. [12pt] Let C(x,y) mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Express each of these statements by a simple English sentence.

a. [2pt] $\mathsf{C}(\text{Randy Goldberg}, \text{CS } 252)$

Randy Goldberg is enrolled in CS-252.

b. [2pt] $\exists x \mathsf{C}(x, \text{Math } 695)$

Someone is enrolled in Math-695.

c. [2pt] $\exists y \mathsf{C}(\text{Carol Sitea}, y)$

Carol Sitea is enrolled in some course.

d. [2pt] $\exists x (\mathsf{C}(x, \text{Math } 222) \land \mathsf{C}(x, \text{Math } 252)$

There is a student who is enrolled in both Math-222 and Math-252.

e. [2pt] $\exists x \exists y \forall z ((x \neq y) \land (\mathsf{C}(x, z) \rightarrow \mathsf{C}(y, z)))$

There are two students such that if the first one is enrolled in a class, the second one is also enrolled in that class.

f. [2pt] $\exists x \exists y \forall z ((x \neq y) \land (\mathsf{C}(x, z) \leftrightarrow \mathsf{C}(y, z)))$

There are two students who are enrolled in all the same classes.