Review Questions

For each of the following languages, answer each of the following three questions, and *prove* your answers are correct.

- Is the language recognizable?
- Is the complement of the language recognizable?
- Is the language decidable?

$L_1 = \{\langle M \rangle : M \text{ is a TM that rejects at least 3 different input strings}\}$.

$L_2 = \{\langle M \rangle : M \text{ is a TM and there exists an input string } w \text{ such that } M \text{ halts within 10 steps on input } w\}$.

$L_3 = \{\langle M, w_1, w_2 \rangle : M \text{ is a TM that accepts neither } w_1 \text{ nor } w_2\}$.

$L_4 = \{\langle M, w_1, w_2 \rangle : M \text{ is a TM that accepts } w_1 \text{ but does not accept } w_2\}$.

$L_5 = \{\langle M \rangle : M \text{ is a TM that accepts every input string}\}$.

$L_6 = \{\langle M, k \rangle : k \in \mathbb{N} \text{ and } M \text{ is a TM that accepts at least } k \text{ different strings}\}$.

$L_7 = \{\langle M, a \rangle : M \text{ is a TM that never writes the character } a \text{ when run on the input string } \varepsilon\}$.

$L_8 = \{\langle M_1, M_2 \rangle : M_1 \text{ and } M_2 \text{ are TMs such that } L(M_1) = \overline{L(M_2)}\}$.