Homework Assignment #4
Due: October 9, 2014 at 4:00 p.m.

1. If L is a language over the alphabet Σ, let $EXTRA(L)$ be the set of all strings obtained by inserting exactly one extra character into any one of the strings in L. More formally, $EXTRA(L) = \{xay : x, y \in \Sigma^* \text{ and } xy \in L \text{ and } a \in \Sigma\}$.

(a) If $\Sigma = \{\mathtt{a}, \mathtt{b}\}$ and $L_1 = \{\mathtt{aa}, \varepsilon, \mathtt{b}\}$, what is $EXTRA(L_1)$?

(b) Does there exist a language L_2 such that $EXTRA(L_2) = L_2$? Prove your answer is correct.

(c) Explain why the set of regular languages is closed under the $EXTRA$ operation. (In other words, show that if L is regular, then $EXTRA(L)$ must also be regular.) Your argument should have the same form as the proof of Theorem 1.47 in the textbook: first give a high-level description of your proof idea in English, then give a detailed description of the construction. In addition, you should describe, for each state of your new machine, exactly which strings will take the machine into that state (but you do not need to give a formal proof of this claim).