Random variables (continued)

Repeated measurements

- student scientists are taught to perform multiple measurements and average the results
  - one reason is to increase the precision of the estimated value
  - if taking multiple measurements increases the precision then surely taking even more measurements would be better?

- how does the precision of the average behave as a function of the number of measurements n?

Variance of the mean

- precision = variance (or standard deviation)
- assume the measurement is contaminated with additive Gaussian noise with mean zero and variance 1
- write a simulation to study the behavior of the sample mean as a function of the number of measurements

Variance of the mean

```
%% Variance of the mean for n = 1, 2, ..., 64
N = 64;
VAR = ones(1, N);
for n = 1:N
    X = randn(n, 10000);
    if n == 1
        VAR(n) = var(X);
    else
        VAR(n) = var(mean(X));
    end
end
plot(VAR)
```

Variance of the mean

- variance of the mean

$$\mathrm{var}(\bar{x}) \propto \frac{1}{n}$$

- standard deviation of the mean

$$\mathrm{stddev}(\bar{x}) \propto \frac{1}{\sqrt{n}}$$

Gaussian random walk

- a Gaussian random walk is a random walk where the step size is drawn from a Gaussian distribution with mean variance

Gaussian random walk

```
%% Gaussian random walk for n = 100 steps
n = 100;
w = zeros(1, n);
for step = 2:n
    w(step) = w(step - 1) + randn(1);
end
plot(w)
```

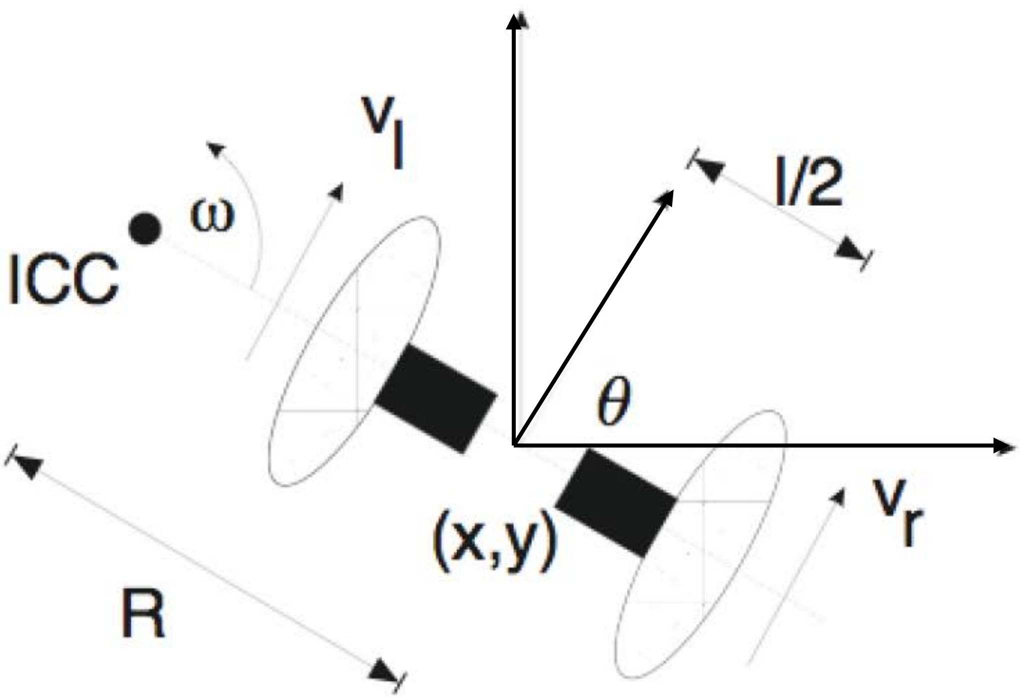
Gaussian random walk

- after *n* steps what does the walk look like?

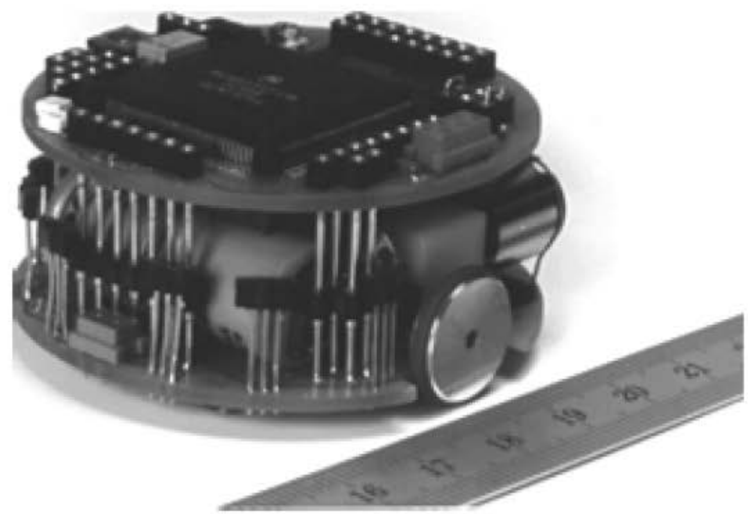
```
%% Many Gaussian random walks
TRIALS = 100000;
W = zeros(TRIALS, n);
tic
for t = 1:TRIALS           % there is faster way to do this...
    for step = 2:n
        W(t, step) = W(t, step - 1) + randn(1);
    end
end
toc
plot(var(W))
```

Differential drive

- two independently driven wheels mounted on a



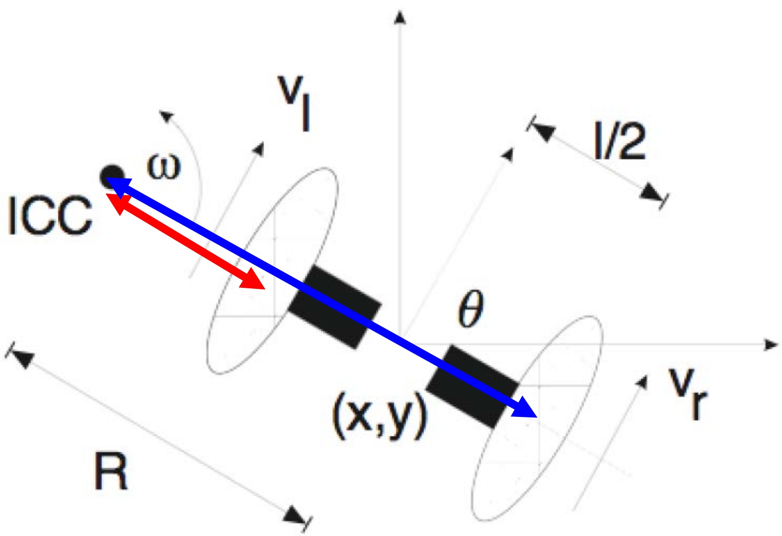
(a) Differential Drive



(b) Khepera Robot

Differential drive

- velocity constraint defines the wheel ground velocities



(a) Differential Drive

instantaneous linear velocity of left wheel

$$v_l = \omega(R + \frac{\ell}{2})$$

$$v_r = \omega(R - \frac{\ell}{2})$$

instantaneous linear velocity of right wheel

Differential drive

- given the wheel ground velocities it is easy to solve for the radius, R, and angular velocity ω

$$R = \frac{\ell}{2}\frac{\left(v_r + v_\ell\right)}{\left(v_r - v_\ell\right)}$$

$$\omega = \frac{\left(v_r - v_\ell\right)}{\ell}$$

- given the radius and angular velocity it is possible to estimate the path of the robot
  - but let's look at a simpler model of the motion instead

Differential drive

- assume at time t the robot has position $\mathbf{p}_t = \begin{bmatrix} x_t \\ y_t \end{bmatrix}$

  orientation θ_t

  and forward velocity v_t

- then after 1 unit of time has passed the new position of the robot is

$$\mathbf{p}_{t+1} = \begin{bmatrix} x_t \\ y_t \end{bmatrix} + v_t \begin{bmatrix} \cos\theta_t \\ \sin\theta_t \end{bmatrix}$$

Differential drive

- suppose that the robot starts at the origin facing right
- suppose that the true values of $\theta_t = 0$ and $v_t = 1$ are constant but the robot has an uncertain estimate of its velocity $\hat{v}_t = v_t + \mathcal{N}(0, 0.25^2)$

normal random variable with mean 0 and variance 0.25^2

- what is the robot's estimate of its position after t=1,2,3, ..., 10 time steps?

Differential drive

```
%% Differential drive (1 trial)
theta = 0;
v = 1;
P = zeros(2, 11);
for t = 2:11
    vhat = v + 0.25*randn(1);
    P(:, t) = P(:, t-1) + vhat * [cos(theta); sin(theta)];
end
plot(P(1, :), P(2, :), '.');
```

Differential drive

```
%% Differential drive (many trials)
TRIALS = 1000;
theta = 0;
v = 1;
Px = zeros(TRIALS, 11);
Py = zeros(TRIALS, 11);
for trial = 1:TRIALS
    for t = 2:11
        pprev = [Px(trial, t - 1); Py(trial, t - 1)];
        vhat = v + 0.25*randn(1);
        p = pprev + vhat*[cos(theta); sin(theta)];
        Px(trial, t) = p(1);
        Py(trial, t) = p(2);
    end
end
hist(Px(:, 11))
```

Differential drive

- suppose that the robot starts at the origin facing right
- suppose that the true values of $\theta_t = 0$ and $v_t = 1$ are constant but the robot has an uncertain estimate of its orientation $\hat{\theta}_t = \theta_t + \mathcal{N}(0, (10\text{deg})^2)$
- what is the robot's estimate of its position after t=1,2,3, …, 10 time steps?

Differential drive

```
%% Differential drive (many trials)
TRIALS = 1000;
theta = 0;
v = 1;
Px = zeros(TRIALS, 11);
Py = zeros(TRIALS, 11);
for trial = 1:TRIALS
    for t = 2:11
        pprev = [Px(trial, t - 1); Py(trial, t - 1)];
        thetahat = theta + (10 * pi/180)*randn(1);
        p = pprev + v*[cos(thetahat); sin(thetahat)];
        Px(trial, t) = p(1);
        Py(trial, t) = p(2);
    end
end
plot(Px(:, 11), Py(:,11), '.')
```