Computing for the Physical Sciences

CSE1541M

Who Am I?

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  - Lassonde 2046
  - hours : see syllabus on course web page
- email
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Course Format

- everything you need to know is on the course website
  - http://www.eecs.yorku.ca/course/1541

- labs start next Tuesday (Jan 14)

CSE1541 Overview

- an introductory programming course using MATLAB
- examples and problems drawn from physics

What is MATLAB?

- a numerical computing environment that has its own programming language
  - interactive: the user can enter commands and "stuff" happens
  - visualization: rich set of plotting functionality
  - programmable: the user can create programs that can be run within the MATLAB environment

A Quick Tour of MATLAB

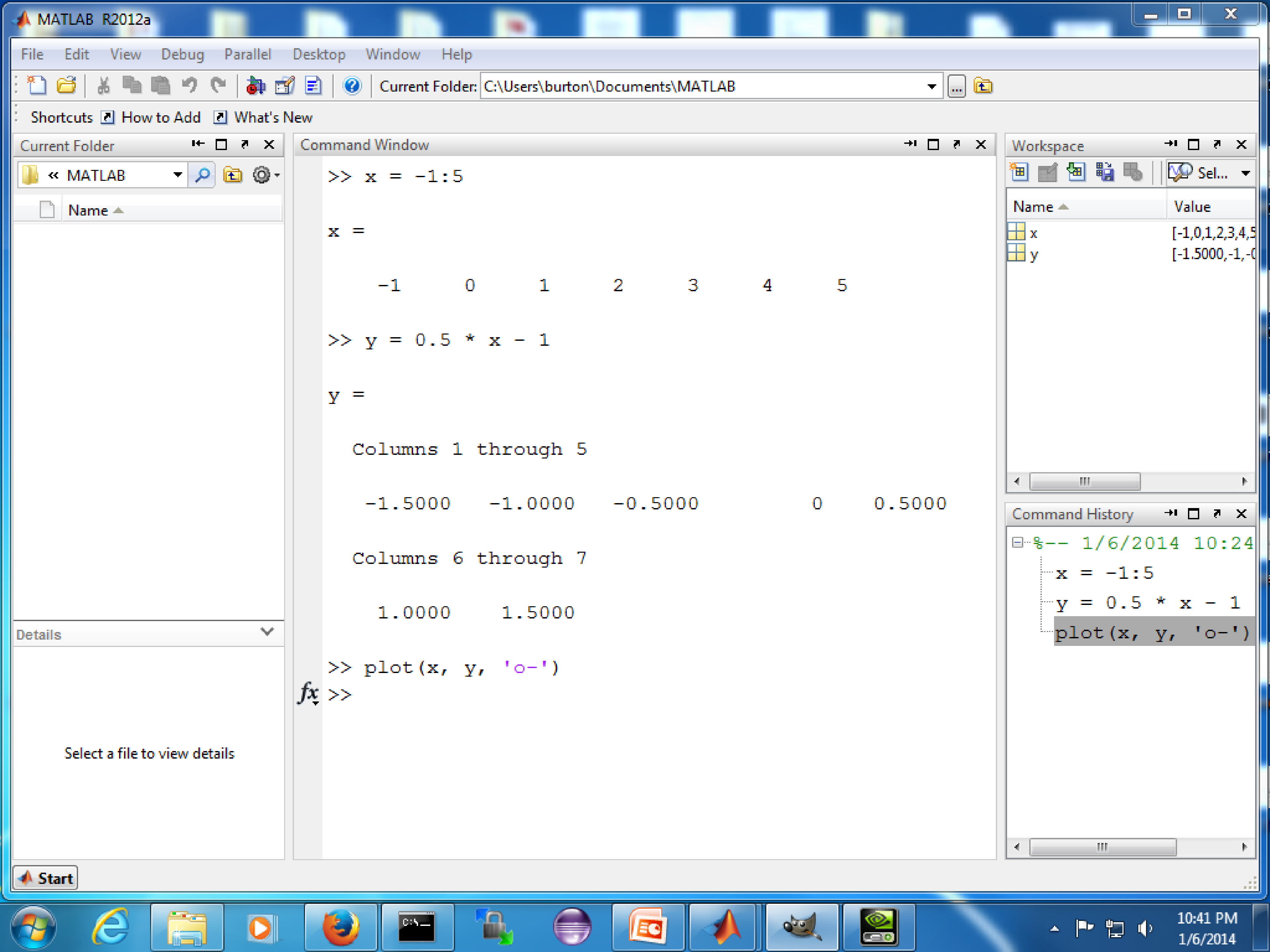
- the equation of a non-vertical line in 2D is:

$$y = mx + b$$

- plot the line

$$y = \frac{1}{2}x - 1$$

on the domain -1 <= x <= 5


MATLAB R2012a
File Edit View Debug Parallel Desktop Window Help
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
MATLAB
Name
Details
Select a file to view details
Command Window
>> x = -1:5
x =
-1 0 1 2 3 4 5
>> y = 0.5 * x - 1
y =
Columns 1 through 5
-1.5000 -1.0000 -0.5000 0 0.5000
Columns 6 through 7
1.0000 1.5000
>> plot(x, y, 'o-')
>>
Workspace
Name Value
x [-1,0,1,2,3,4,5
y [-1.5000,-1,-0
Command History
%-- 1/6/2014 10:24
x = -1:5
y = 0.5 * x - 1
plot(x, y, 'o-')
Start
10:41 PM
1/6/2014

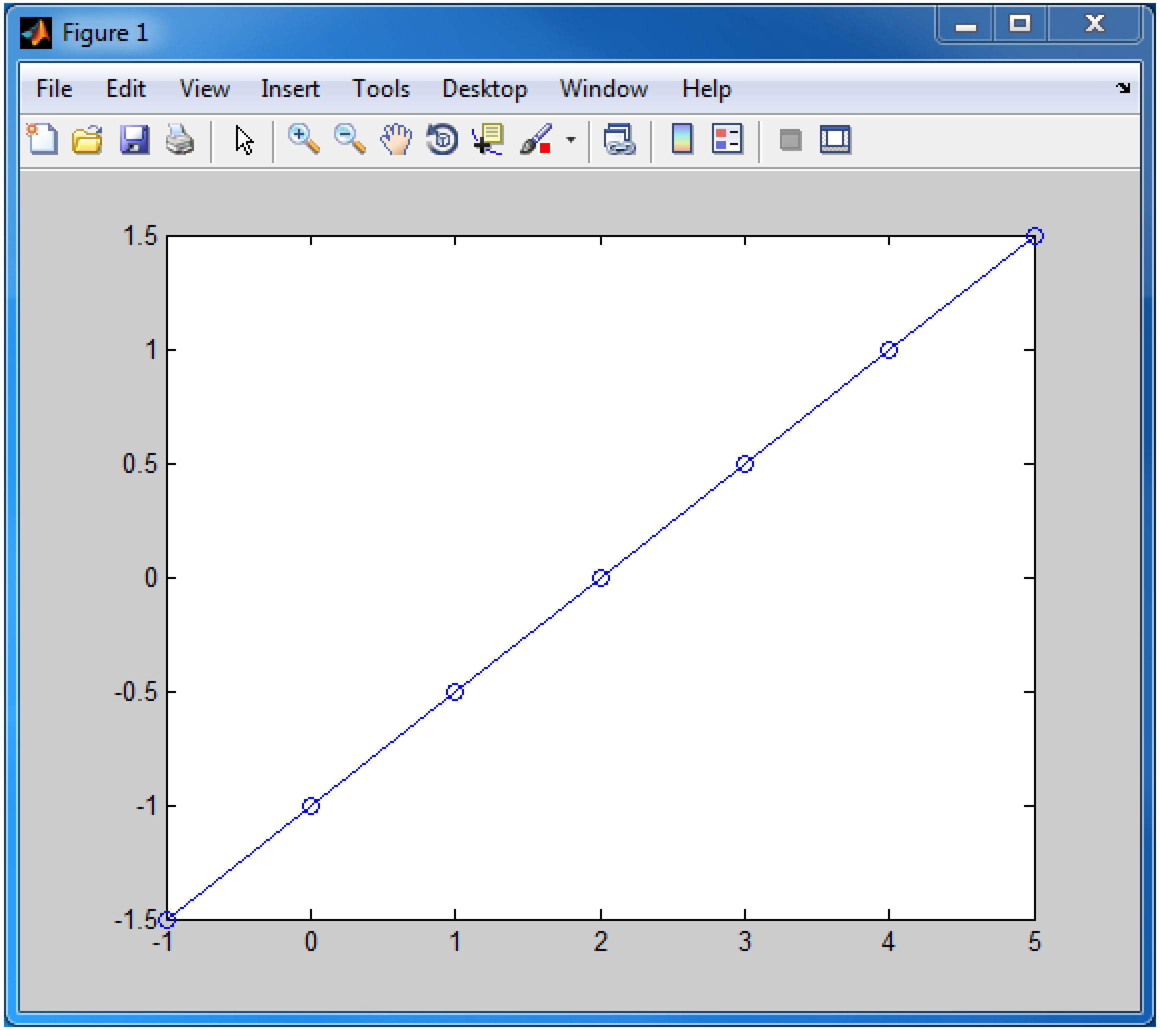


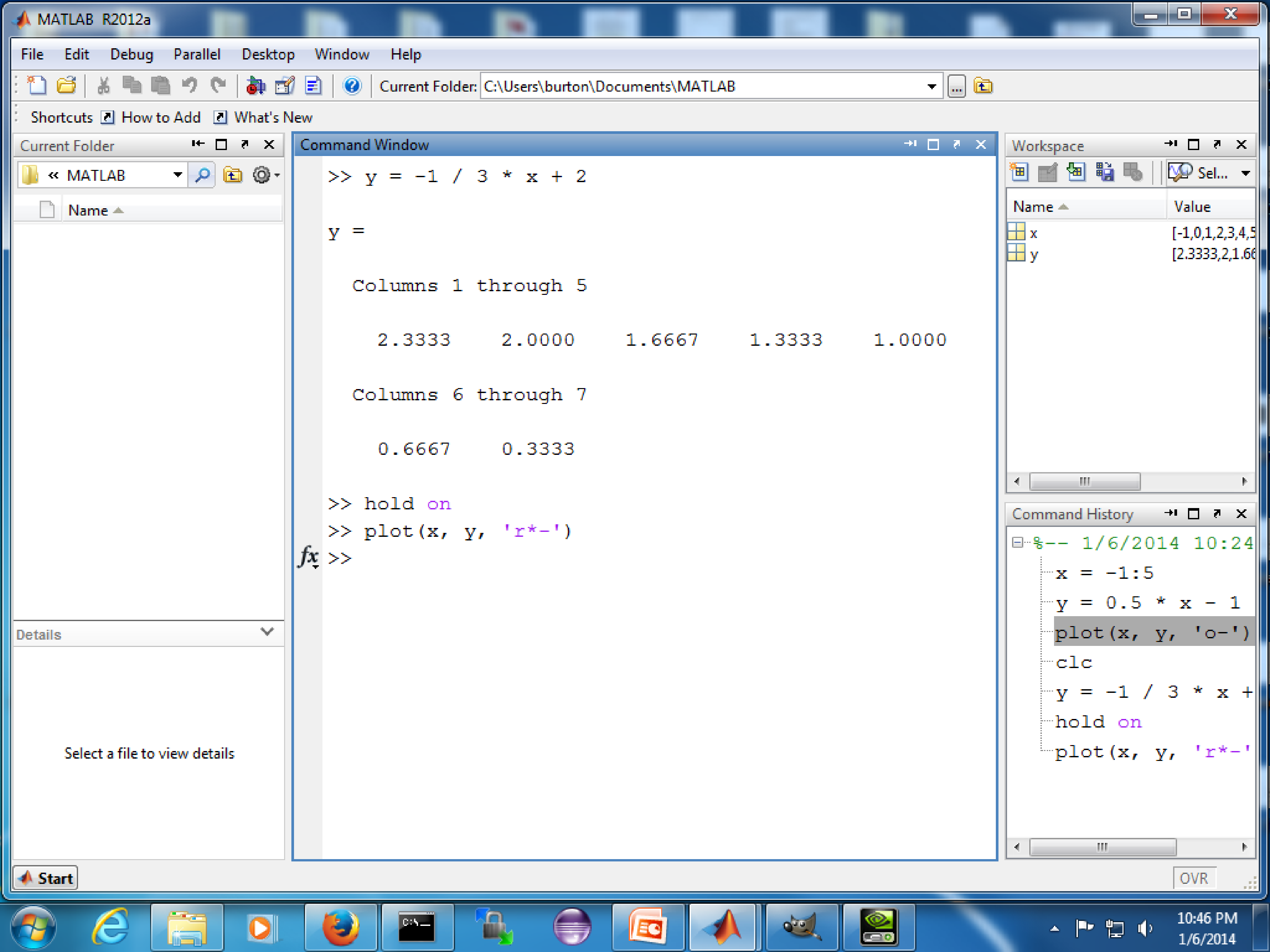
Figure 1
File Edit View Insert Tools Desktop Window Help
1.5
1
0.5
0
-0.5
-1
-1.5
-1 0 1 2 3 4 5

A Quick Tour of MATLAB

- find the intersection of the two lines:

$$y = \tfrac{1}{2}x - 1$$

$$y = -\tfrac{1}{3}x + 2$$


MATLAB R2012a
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
« MATLAB
Name
Details
Select a file to view details
Command Window
>> y = -1 / 3 * x + 2
y =
Columns 1 through 5
2.3333 2.0000 1.6667 1.3333 1.0000
Columns 6 through 7
0.6667 0.3333
>> hold on
>> plot(x, y, 'r*-')
>>
Workspace
Sel...
Name Value
x [-1,0,1,2,3,4,5
y [2.3333,2,1.66
Command History
%-- 1/6/2014 10:24
x = -1:5
y = 0.5 * x - 1
plot(x, y, 'o-')
clc
y = -1 / 3 * x +
hold on
plot(x, y, 'r*-'
Start
OVR
10:46 PM
1/6/2014

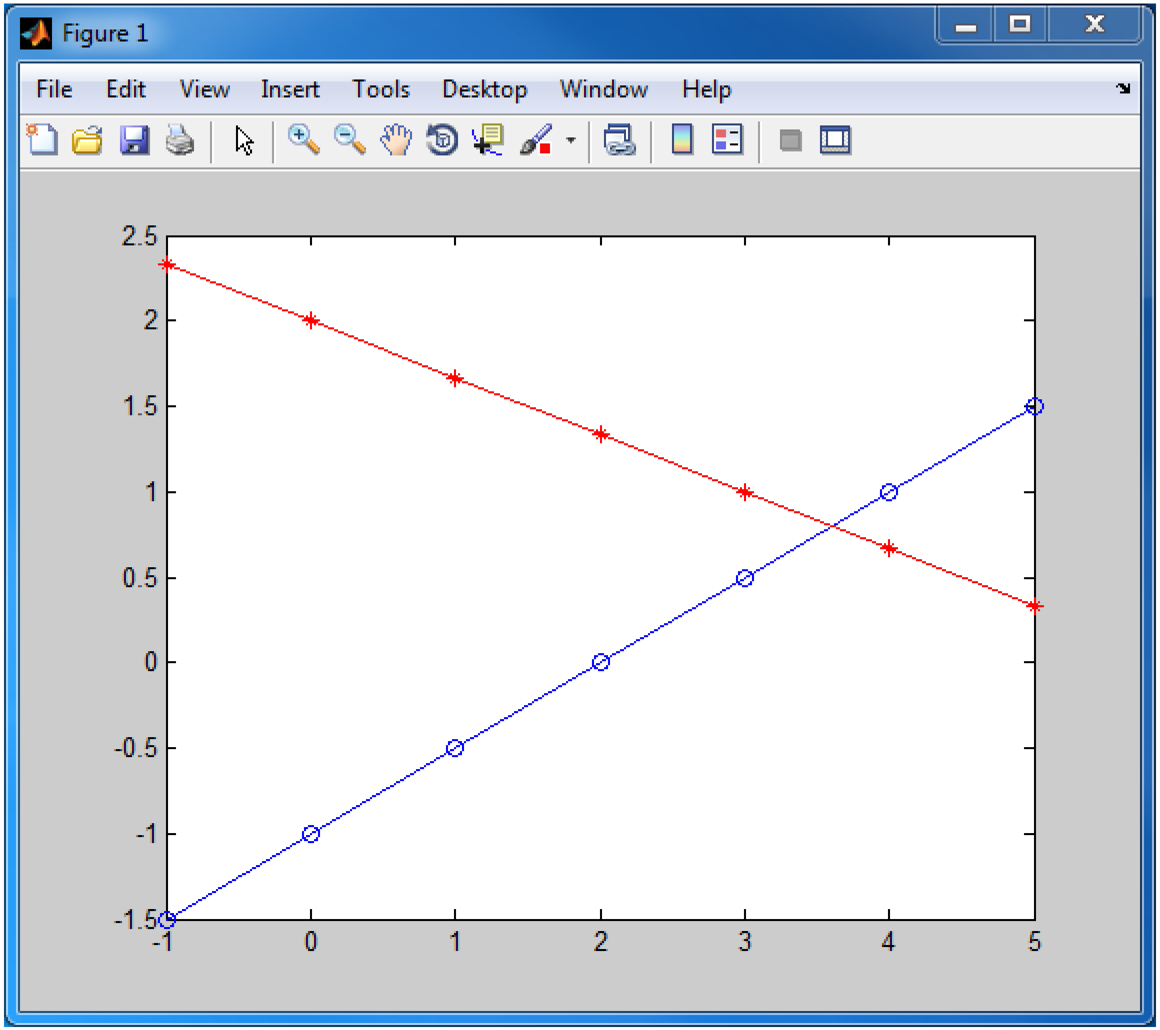


Figure 1
File Edit View Insert Tools Desktop Window Help
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-1
0
1
2
3
4
5

A Quick Tour of MATLAB

- it looks like the intersection point is somewhere around

$$\begin{bmatrix} 3.7 \\ 0.7 \end{bmatrix}$$

- can we find the exact intersection point?

A Quick Tour of MATLAB

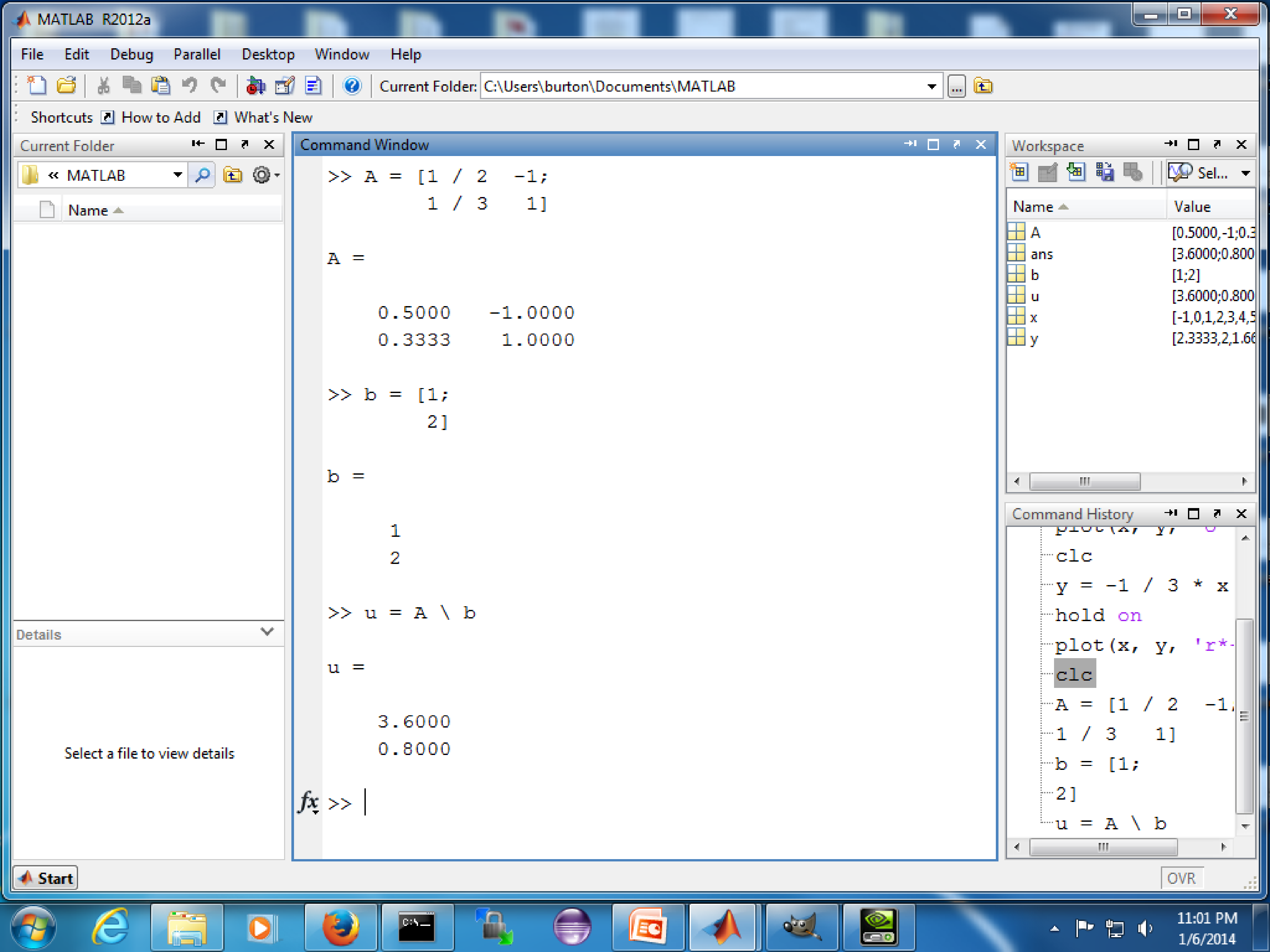
- rewrite the equations of the lines as:

$$\tfrac{1}{2}x - y = 1$$

$$\tfrac{1}{3}x + y = 2$$

- this system of two equations can be written in matrix form as:

$$\begin{bmatrix} \frac{1}{2} & -1 \\ \frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$


MATLAB R2012a
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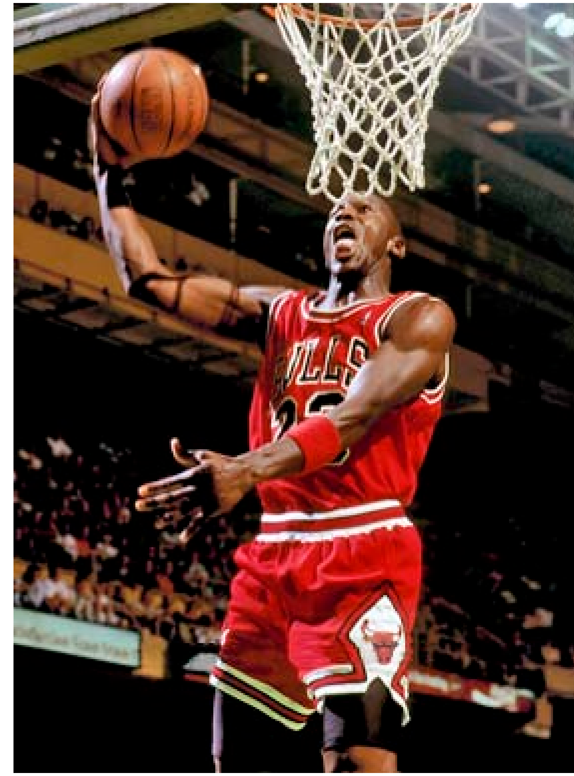
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
MATLAB
Name
Details
Select a file to view details
Command Window
>> A = [1 / 2 -1;
1 / 3 1]
A =
0.5000 -1.0000
0.3333 1.0000
>> b = [1;
2]
b =
1
2
>> u = A \ b
u =
3.6000
0.8000
>>
Workspace
Name Value
A [0.5000,-1;0.3
ans [3.6000;0.800
b [1;2]
u [3.6000;0.800
x [-1,0,1,2,3,4,5
y [2.3333,2,1.66
Command History
clc
y = -1 / 3 * x
hold on
plot(x, y, 'r*-
clc
A = [1 / 2 -1;
1 / 3 1]
b = [1;
2]
u = A \ b
Start
OVR
11:01 PM
1/6/2014

A Quick Tour of MATLAB



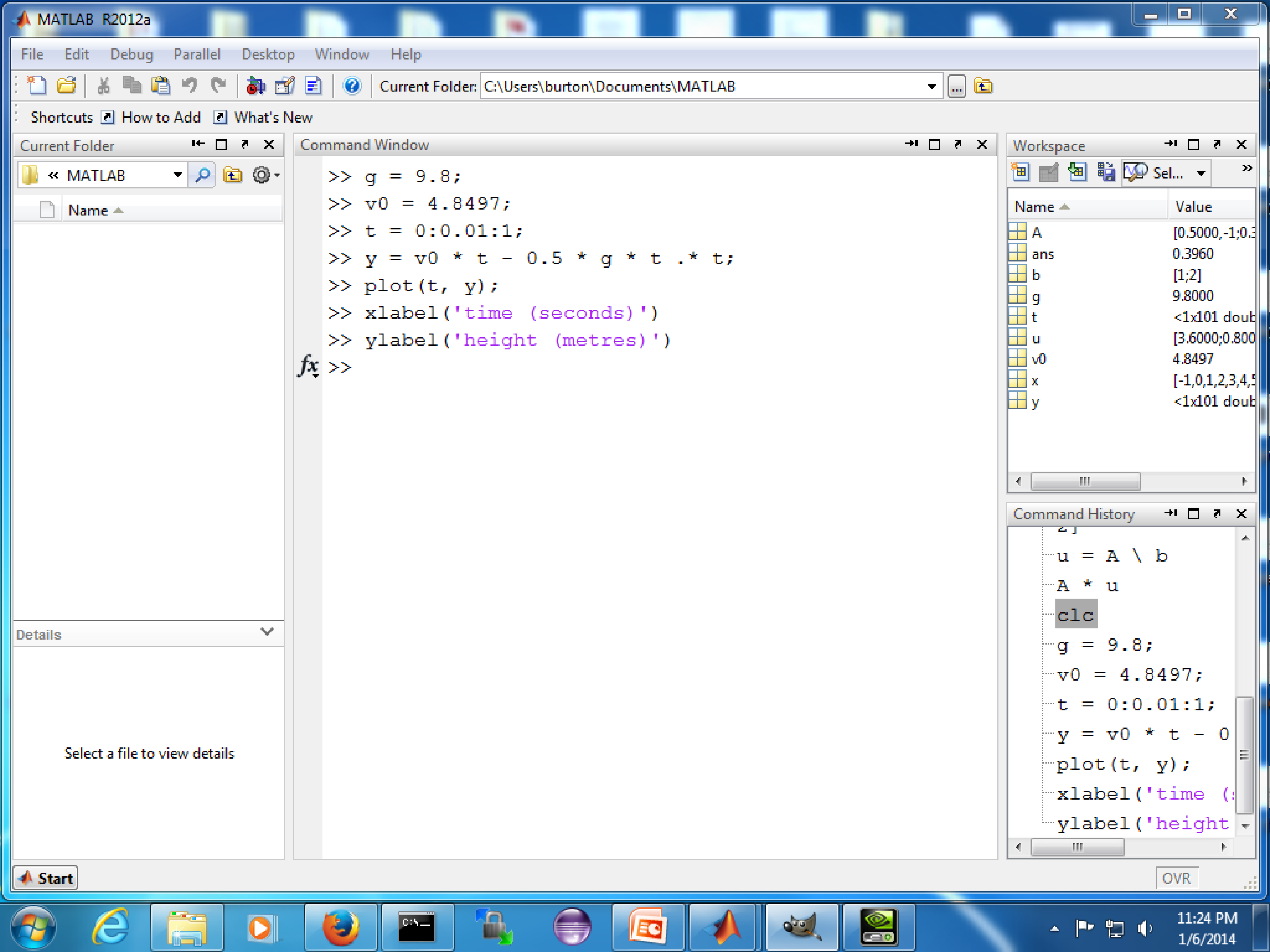
- elite basketball players seemingly defy gravity by hanging in the air
- in his prime, Michael Jordan's (MJ) vertical leap was approximately 1.2 m. Assuming $g = 9.8\ m/s^2$, MJ would have to jump vertically with an initial velocity $v_o = 4.8497\ m/s$ to achieve a maximum height of 1.2m
- explain why elite jumpers appear to hang in midair

A Quick Tour of MATLAB

- from the equations of projectile motion, we know that the vertical displacement of the jumper is given by:

$$y(t) = v_0 t - \tfrac{1}{2} g t^2$$

- let's plot $y(t)$ for 0 <= t <= 1


MATLAB R2012a
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
« MATLAB
Name
Details
Select a file to view details
Command Window
>> g = 9.8;
>> v0 = 4.8497;
>> t = 0:0.01:1;
>> y = v0 * t - 0.5 * g * t .* t;
>> plot(t, y);
>> xlabel('time (seconds)')
>> ylabel('height (metres)')
fx >>
Workspace
Name Value
A [0.5000,-1;0.3
ans 0.3960
b [1;2]
g 9.8000
t <1x101 doub
u [3.6000;0.800
v0 4.8497
x [-1,0,1,2,3,4,5
y <1x101 doub
Command History
u = A \ b
A * u
clc
g = 9.8;
v0 = 4.8497;
t = 0:0.01:1;
y = v0 * t - 0
plot(t, y);
xlabel('time (
ylabel('height
Start
OVR
11:24 PM
1/6/2014

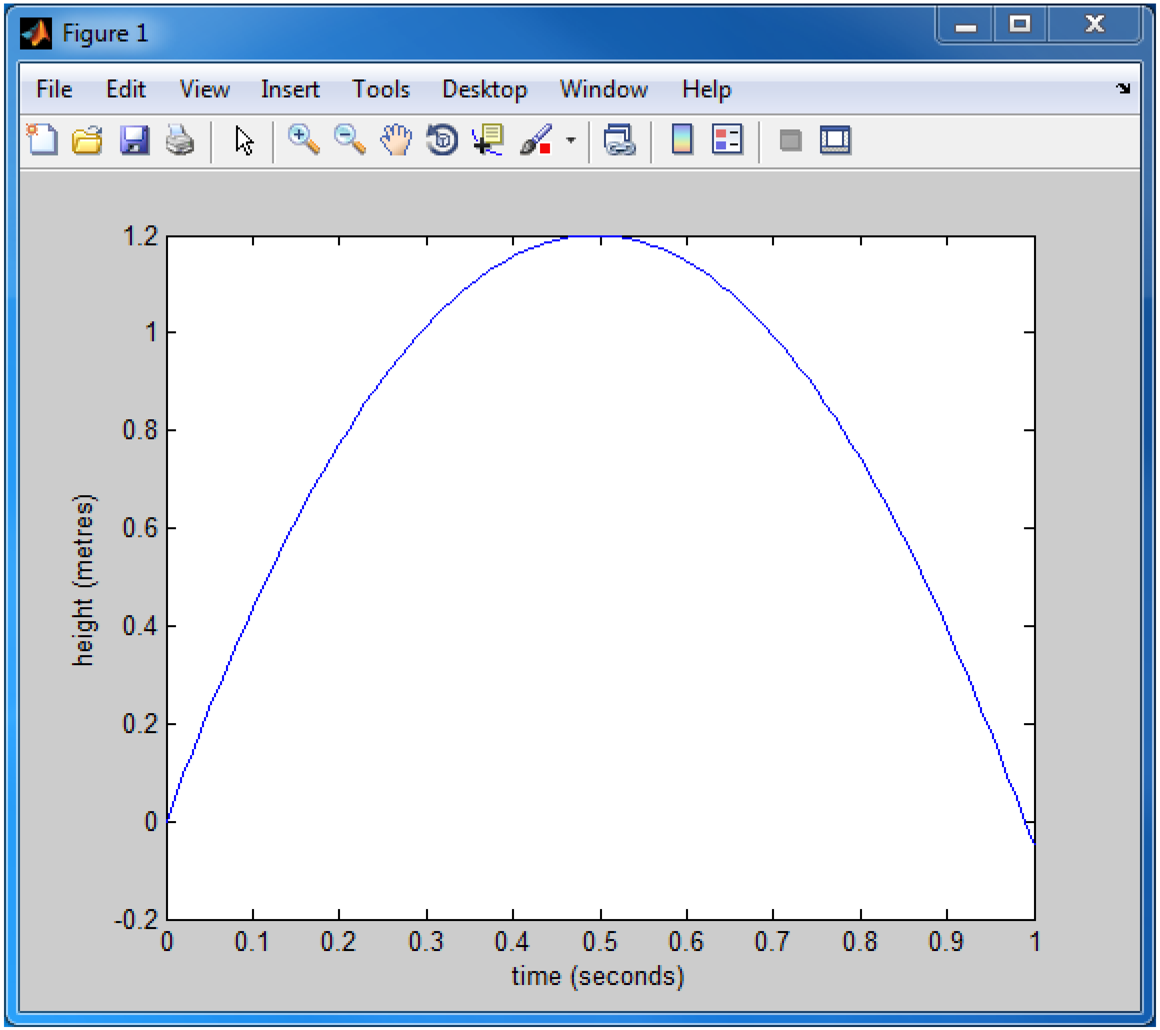
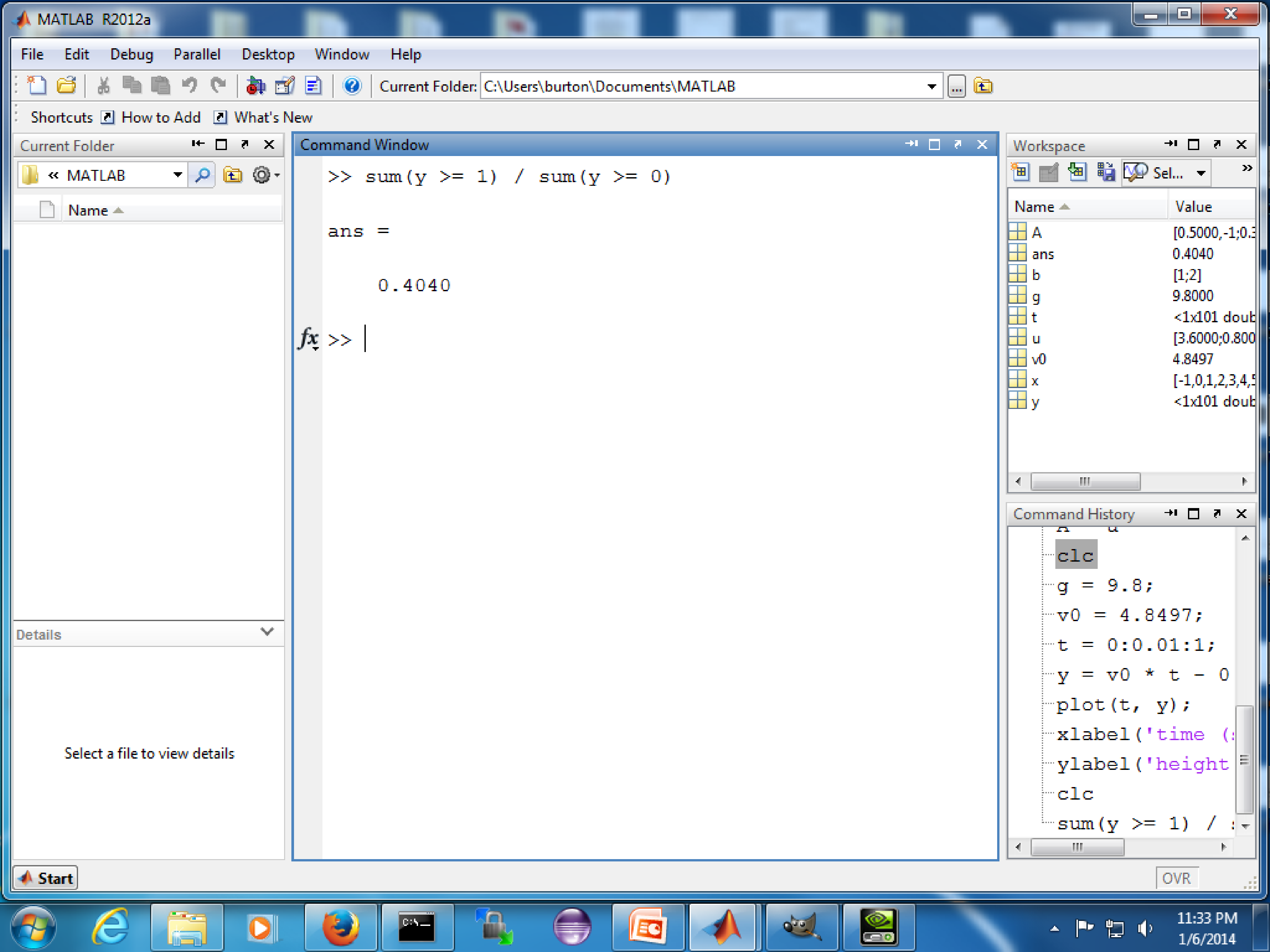


Figure 1
File Edit View Insert Tools Desktop Window Help
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
height (metres)
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1
time (seconds)

A Quick Tour of MATLAB

- this still doesn't really explain why the jumper seems to hang mid-air
- what fraction of the total time spent in the air is the jumper at a height of 1m or more?

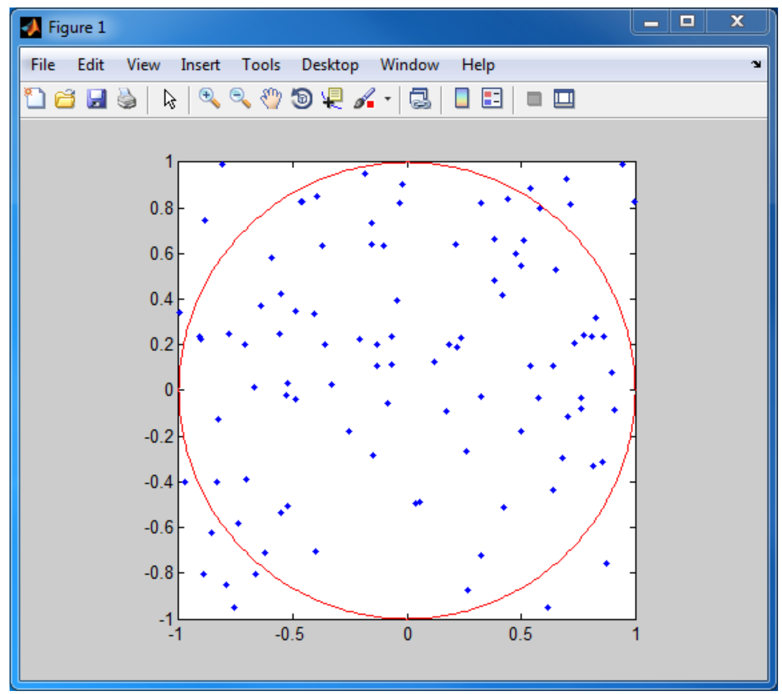
- we could solve this exactly using the quadratic equation
- we could estimate this by counting the number of values of y where $y >= 1$

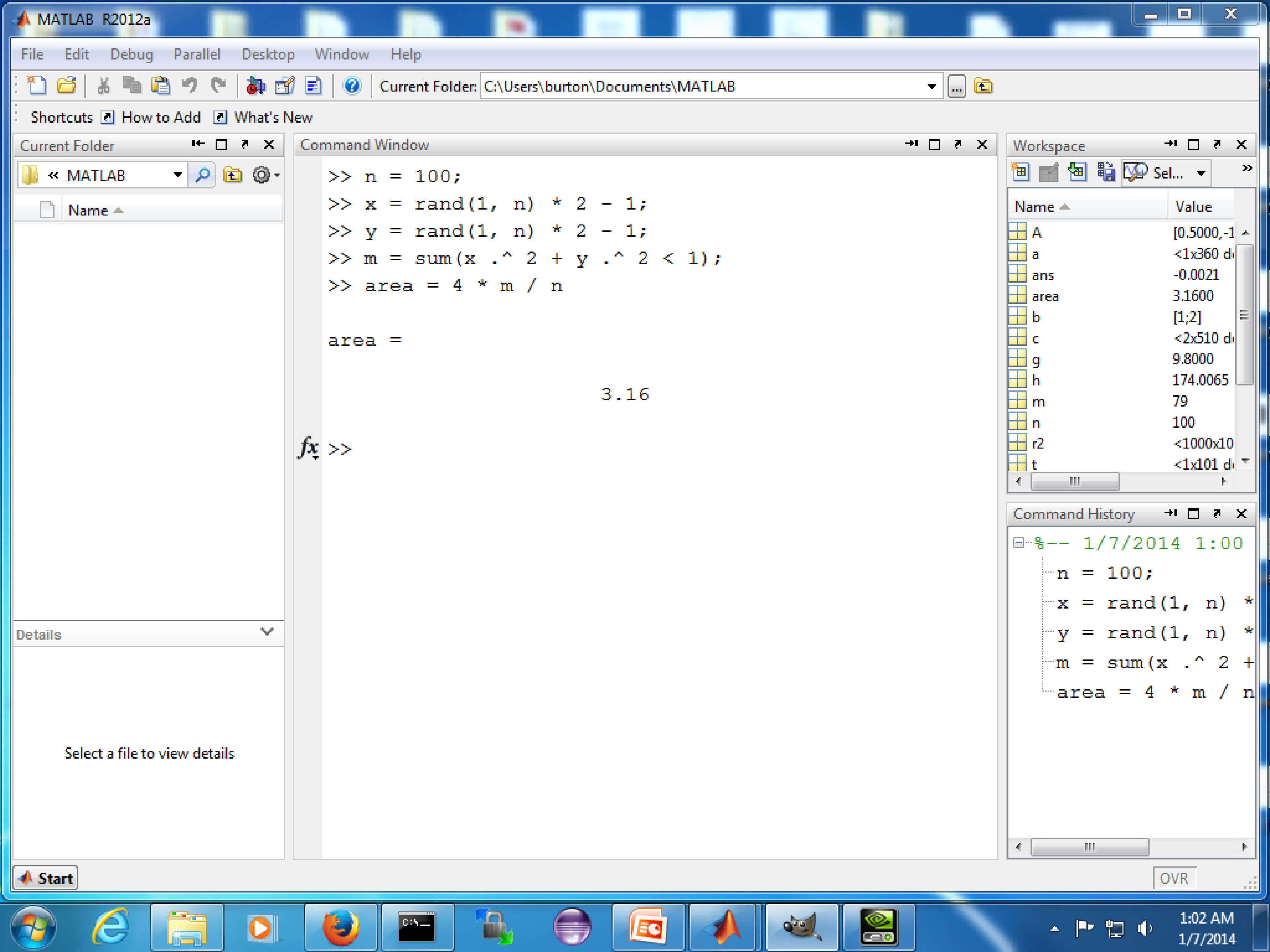

MATLAB R2012a
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
MATLAB
Name
Details
Select a file to view details
Command Window
>> sum(y >= 1) / sum(y >= 0)
ans =
0.4040
>>
Workspace
Name Value
A [0.5000,-1;0.3
ans 0.4040
b [1;2]
g 9.8000
t <1x101 doub
u [3.6000;0.800
v0 4.8497
x [-1,0,1,2,3,4,5
y <1x101 doub
Command History
clc
g = 9.8;
v0 = 4.8497;
t = 0:0.01:1;
y = v0 * t - 0
plot(t, y);
xlabel('time (
ylabel('height
clc
sum(y >= 1) /
Start
OVR
11:33 PM
1/6/2014

A Quick Tour of MATLAB

- Monte Carlo integration is a technique for numerical integration that uses random numbers
- a classic example is calculating the area of a circle of radius 1

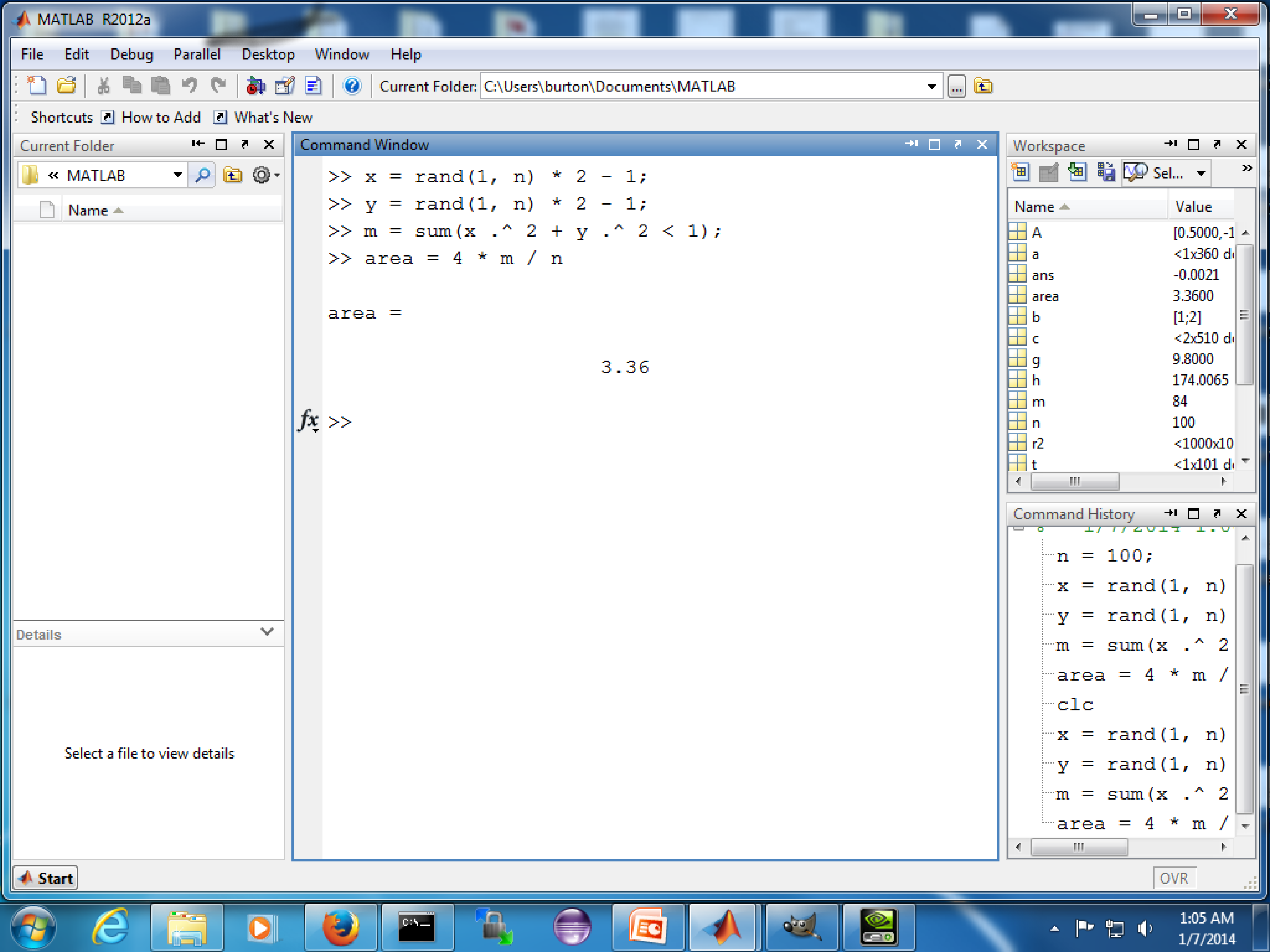
1. generate *n* random points inside the square containing the circle
2. count the number of points *m* inside the circle
3. estimate the area as 4 * *m* / *n*




MATLAB R2012a
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
MATLAB
Name
Details
Select a file to view details
Command Window
>> n = 100;
>> x = rand(1, n) * 2 - 1;
>> y = rand(1, n) * 2 - 1;
>> m = sum(x .^ 2 + y .^ 2 < 1);
>> area = 4 * m / n
area =
3.16
>>
Workspace
Sel...
Name Value
A [0.5000,-1
a <1x360 d
ans -0.0021
area 3.1600
b [1;2]
c <2x510 d
g 9.8000
h 174.0065
m 79
n 100
r2 <1000x10
t <1x101 d
Command History
%-- 1/7/2014 1:00
n = 100;
x = rand(1, n) *
y = rand(1, n) *
m = sum(x .^ 2 +
area = 4 * m / n
Start
OVR
1:02 AM
1/7/2014

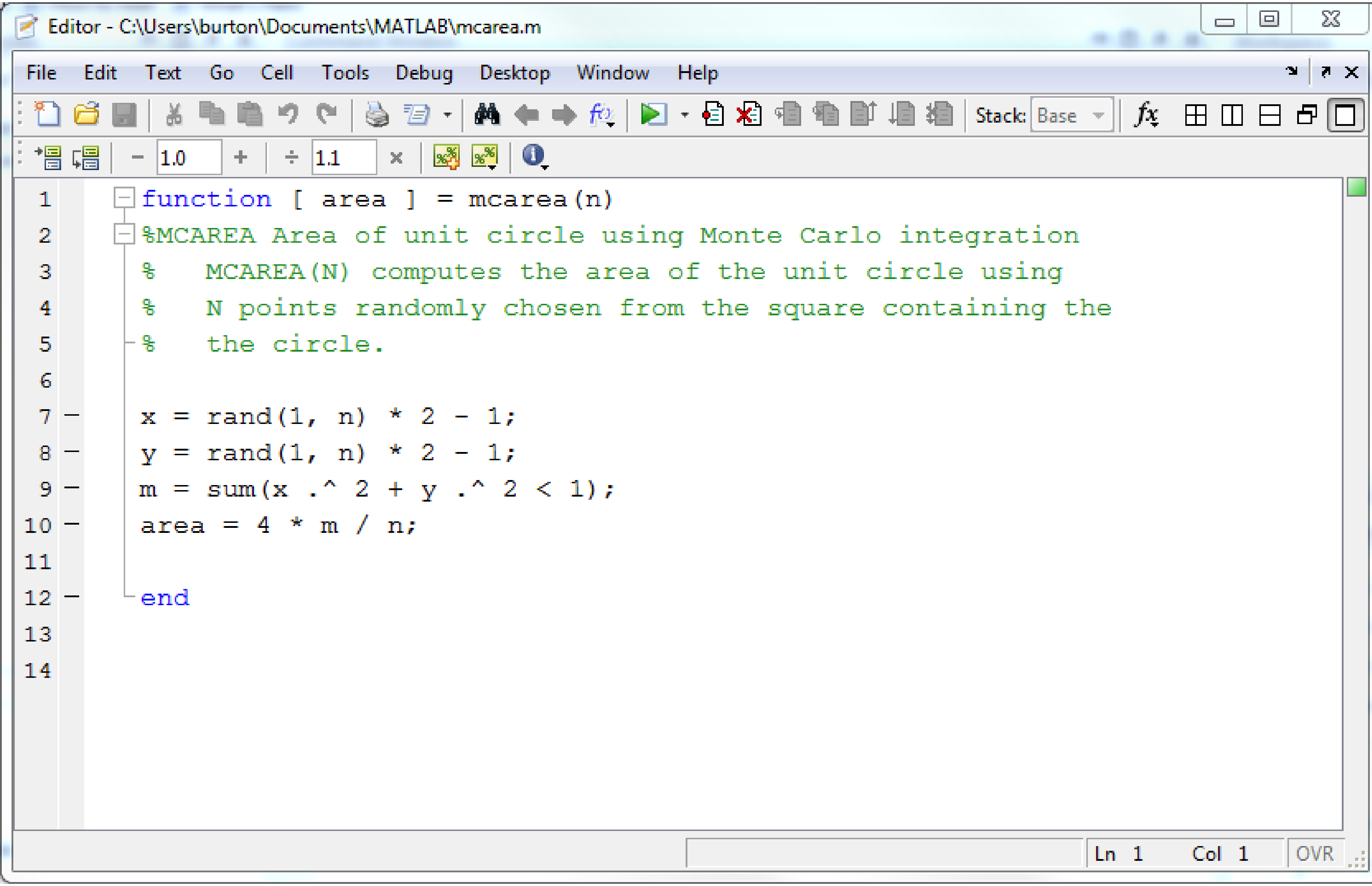
A Quick Tour of MATLAB

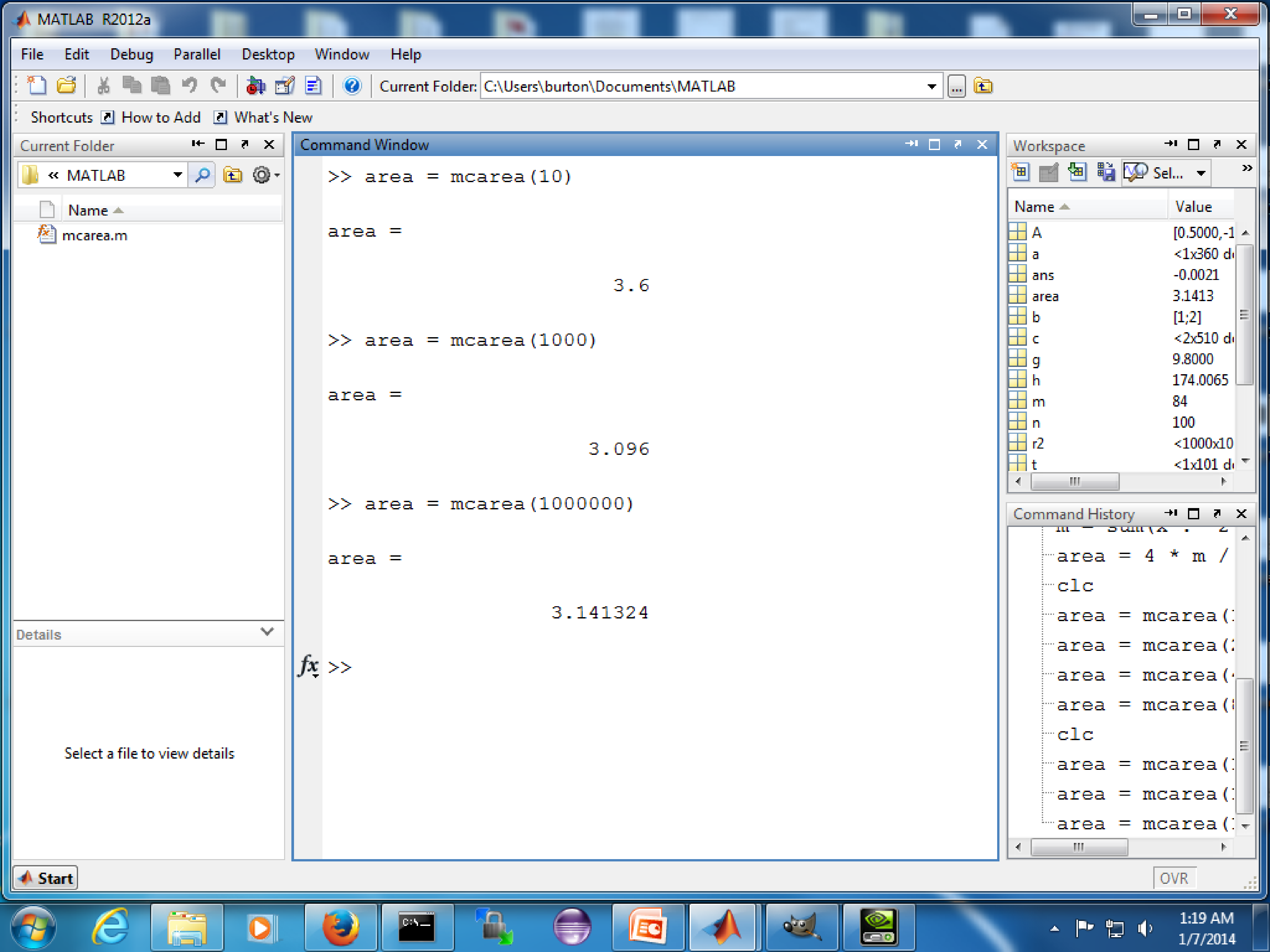
- if you repeat the process, you will probably get a different answer
  - because the points are chosen at random


MATLAB R2012a
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
« MATLAB
Name
Details
Select a file to view details
Command Window
>> x = rand(1, n) * 2 - 1;
>> y = rand(1, n) * 2 - 1;
>> m = sum(x .^ 2 + y .^ 2 < 1);
>> area = 4 * m / n
area =
3.36
fx >>
Workspace
Sel...
Name Value
A [0.5000,-1
a <1x360 d
ans -0.0021
area 3.3600
b [1;2]
c <2x510 d
g 9.8000
h 174.0065
m 84
n 100
r2 <1000x10
t <1x101 d
Command History
n = 100;
x = rand(1, n)
y = rand(1, n)
m = sum(x .^ 2
area = 4 * m /
clc
x = rand(1, n)
y = rand(1, n)
m = sum(x .^ 2
area = 4 * m /
Start
OVR
1:05 AM
1/7/2014

A Quick Tour of MATLAB

- we might want to repeat the calculation many times to find out:
  - how much the estimate varies for a given value of n
  - how accurate the estimate is for a given value of n
  - how the precision and accuracy vary as a function of n
- to repeat a calculation made up of several commands you can put the commands in a user-defined function
  - you (or anyone else) can then call the function with a single command


Editor - C:\Users\burton\Documents\MATLAB\mcarea.m
File Edit Text Go Cell Tools Debug Desktop Window Help
Stack: Base
1.0
1.1
function [area] = mcarea(n)
%MCAREA Area of unit circle using Monte Carlo integration
%   MCAREA(N) computes the area of the unit circle using
%   N points randomly chosen from the square containing the
%   the circle.
x = rand(1, n) * 2 - 1;
y = rand(1, n) * 2 - 1;
m = sum(x .^ 2 + y .^ 2 < 1);
area = 4 * m / n;
end
Ln 1 Col 1 OVR


MATLAB R2012a
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Users\burton\Documents\MATLAB
Shortcuts How to Add What's New
Current Folder
MATLAB
Name
mcarea.m
Details
Select a file to view details
Command Window
>> area = mcarea(10)
area =
3.6
>> area = mcarea(1000)
area =
3.096
>> area = mcarea(1000000)
area =
3.141324
>>
Workspace
Name
Value
A
[0.5000,-1
a
<1x360 d
ans
-0.0021
area
3.1413
b
[1;2]
c
<2x510 d
g
9.8000
h
174.0065
m
84
n
100
r2
<1000x10
t
<1x101 d
Command History
area = 4 * m /
clc
area = mcarea(
area = mcarea(
area = mcarea(
area = mcarea(
clc
area = mcarea(
area = mcarea(
area = mcarea(
Start
OVR
1:19 AM
1/7/2014