You have 2 hours to complete this test.

There are 5 multi-part questions in this test. Create a script named `test1.m` to answer the questions. For each part of each question, create a new cell (section) in your script. You do not need to publish a report from your script.

You have access to the internet during this test. Please do not use any communication software such as email, text chat, social media services, etc.

When you are finished your test, submit your script using the online submit service `https://webapp.eecs.yorku.ca/submit/`

1. (9 points)

   (a) (1 point) Create a row vector named `x` containing the 11 elements $-5, -4, -3, ..., 4, 5$. Avoid typing in all 11 elements individually.

   (b) (1 point) Show how to get the value of the third element of `x`.

   (c) (1 point) Show how to change the value of the third element of `x` to -1000; this should cause the vector `x` to change.

   (d) (1 point) Show how to get the value of the last element of `x`.

   (e) (1 point) Show how to change the value of the last element of `x` to $+1000$; this should cause the vector `x` to change.

   (f) (1 point) Show how to remove the second last elment of `x`; this should cause the vector `x` to change.

   (g) (1 point) Show how to insert the new element -6 to the front of `x`; this should cause the vector `x` to change.

   (h) (1 point) Show how to insert the new element with value 1.5 between the elements with values 1 and 2 in `x`; this should cause the vector `x` to change.

   (i) (1 point) After doing 1(f)–(h), show how to get the number of elements in `x`.

2. (7 points)

(a) (1 point) Create a column vector named `y` containing the 11 elements $50, 40, 30, ..., -40, -50$. Avoid typing in all 11 elements individually.

(b) (2 points) Show how to get all of the elements of `y` that start with an even number (i.e., get every second element of `y` starting from the second element; you do not need code that actually checks if the element starts with an odd number).

(c) (2 points) Show how to change all of the elements of `y` that start with an even number; change the values of the elements by adding 1 to them. This should cause the vector `y` to change.

(d) (2 points) After doing 2(c) show how to remove all of the elements of `y` that end with 0 (again, you do not need code that actually checks if the element ends with 0). This should cause the vector `y` to change.

3. (14 points) Create two column vectors named `m` and `f` having the following values:

$$m = \begin{bmatrix} -4 \\ 3.7 \\ 0 \\ 2.5 \\ 1.2 \\ -2.8 \\ -1.4 \end{bmatrix} \qquad f = \begin{bmatrix} -37 \\ 38 \\ 0 \\ 29 \\ 21 \\ -21 \\ -8 \end{bmatrix}$$

(a) (3 points) Create the matrix named `X` where the first column is the vector `m` and the second column has elements that are all one. If you can, avoid typing in seven ones to construct the second column.

(b) (3 points) Create the matrix named `A` that is equal to the matrix product:

$$A = X^T X$$

where X is your matrix `X`. The matrix X^T is the transpose of X.

(c) (1 point) Show how to find the number of rows in the matrix `A` (the answer should be 2 but your solution should work for any matrix `A`).

(d) (1 point) Show how to find the number of columns in the matrix `A` (the answer should be 2 but your solution should work for any matrix `A`).

(e) (2 points) Create the matrix named `B` that is equal to the element-by-element product of `X` with itself. After doing so, if you enter the command `sum(B)`, you should get a 1×2 row vector where the elements have the same value as the diagonal elements of `A`.

(f) (2 points) Create the column vector `y` that is equal to the matrix-vector product:

$$y = X^T f$$

where X and f are your variables `X` and `f`.

(g) (2 points) Solve the linear system of equations for the vector x:

$$Ax = y$$

using your matrix `A` and vector `y`. You may solve the system of equations any way that you want (e.g., using Gaussian elimination, or computing the inverse of `A`, or using some MATLAB operator). The first element of x should be approximately equal to g, the acceleration due to gravity on Earth. The vectors f and m are related by the equation $f = mg$ where f represents noisy measurements of force for masses in m. The elements of x are the slope and y-intercept of the line of best fit of the measurements f and control variable m.

4. (10 points) The normalized sinc function is defined as:

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

The sinc function is very important in signal processing applications (which you may encounter when acquiring measurements of a time-varying system) and in the branch of applied mathemtatics called information theory which has attracted considerable attention from physicists.

Plot the sinc function and the function $\cos(\pi x)$ as a function of x on the same graph for 175 equally spaced values of x in the range -5 to 5. Label the x-axis of the graph. Include a legend that indicates which curve is the sinc function and which curve is the cosine function.

5. (10 points) The following are all short-answer written questions. Include your answers in your script as comments. In MATLAB you can create a comment that spans multiple lines like so:

```
%{
    This
    is
    a multiline comment.
%}
```

(a) (2 points) Suppose that you are working in a base-4 **unsigned** number system where the digits are 0, 1, 2, and 3. Consider the base-4 number 2311. What is the decimal (base-10) value of the base-4 number? Show how you calculated your answer.

(b) (2 points) Consider the **signed** binary number 1011. What is the decimal (base-10) value of the binary number? Show how you calculated your answer.

(c) (2 points) In mathematics, the value of $\cos(\pi/2)$ is exactly 0. Explain why the MATLAB expression:

```
cos(pi / 2) == 0
```

outputs 0 (or false) instead of 1. There are actually at least two different (but related) reasons why the output is 0 instead of 1.

(d) (2 points) By now, you should have learned that comparing two floating point numbers for equality using the `==` operator in MATLAB is unreliable. A better way to compare to numbers `x` and `y` for equality is the following:

```
abs(x - y) < EPS
```

where `abs` is the MATLAB function for absolute value and `EPS` is equal to some small value (say 10^{-6}, for example). If the above expression is true, then the values `x` and `y` are equal to within `EPS`. What problems can you identify if you use this approach? Try to identify two problems if you can.

(e) (2 points) Your fellow student has written a script that calculates a sinusoidal wave $y(t) = A\sin(2\pi ft)$:

```
% amplitude
    A= 1;

% frequency
z =5;

% 100 equally spaced time steps between 0 and 1
t=[0:0.01:1];

% sinusoidal wave
sin = A * sin(2 * pi * z * t);
```

The script seems to run fine, but there are many stylistic errors. List 4 style errors that the student has made.