1. Show that the distance between points is unchanged by rotation; that is $\|p_1 - p_2\| = \|Rp_1 - Rp_2\|$.

2. Suppose that A is a 2×2 matrix where $A^T A = I$ and $\det A = 1$. Show that there exists a unique θ such that

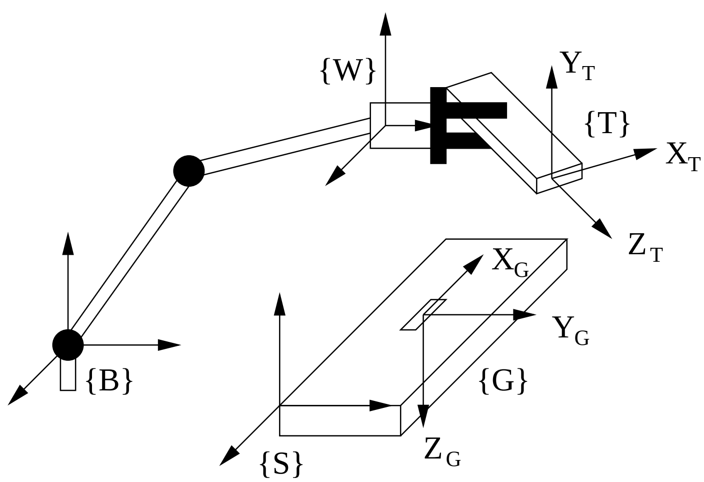
$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

3. (a) Consider all pairs of rotations and translations along the three principle axes, i.e., $R_x R_x$, $R_x R_y$, ..., $R_x T_x$, ..., $T_z T_z$. Which pairs commute?

   (b) Given your answer to (a), what other representations are there for the Denavit-Hartenberg transformation?

4. (a) Suppose you have a frame $\{A\}$ and a frame $\{B\}$. The 4×4 homogeneous matrix T_B^A, where the upper-left 3×3 sub-matrix is a rotation matrix, has three distinct interpretations. What are the interpretations?

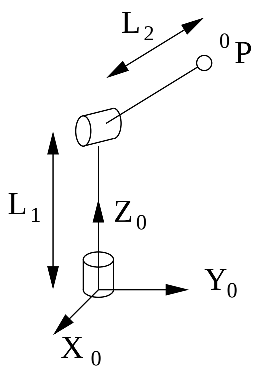
   (b) Consider the figure shown below. The pose of the tool relative to the wrist, T_T^W, is not known. By limping the arm joints, the tool tip can be inserted into the socket, or goal, at location T_G^S. In this calibration configuration, frames $\{G\}$ and $\{T\}$ are coincident, and the pose of the wrist relative to the base, T_W^B, can be retrieved from the robot. Assuming T_S^B and T_G^S are known, give the transform equation to compute the unknown pose of the tool, T_T^W.



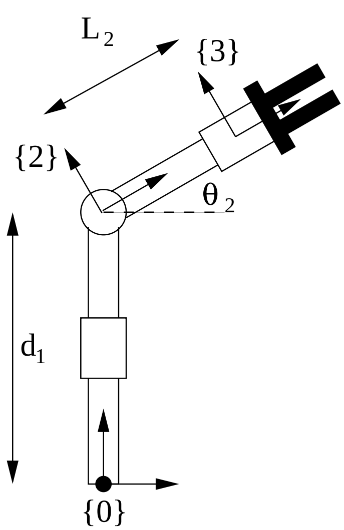
5. (a) What is the forward kinematics problem for a robotic arm?

   (b) What is the inverse kinematics problem for a robotic arm?

   (c) Consider the RR robot (shown below), that is similar to a robot made up of the waist and shoulder joints of the A150 robot. Given a point ${}^0P = \begin{bmatrix} x & y & z \end{bmatrix}^T$ known to be in the workspace of the robot what are the joint angles θ_1 and θ_2? Assume that θ_1 is measured from X_0 and θ_2 is measured from the horizon.



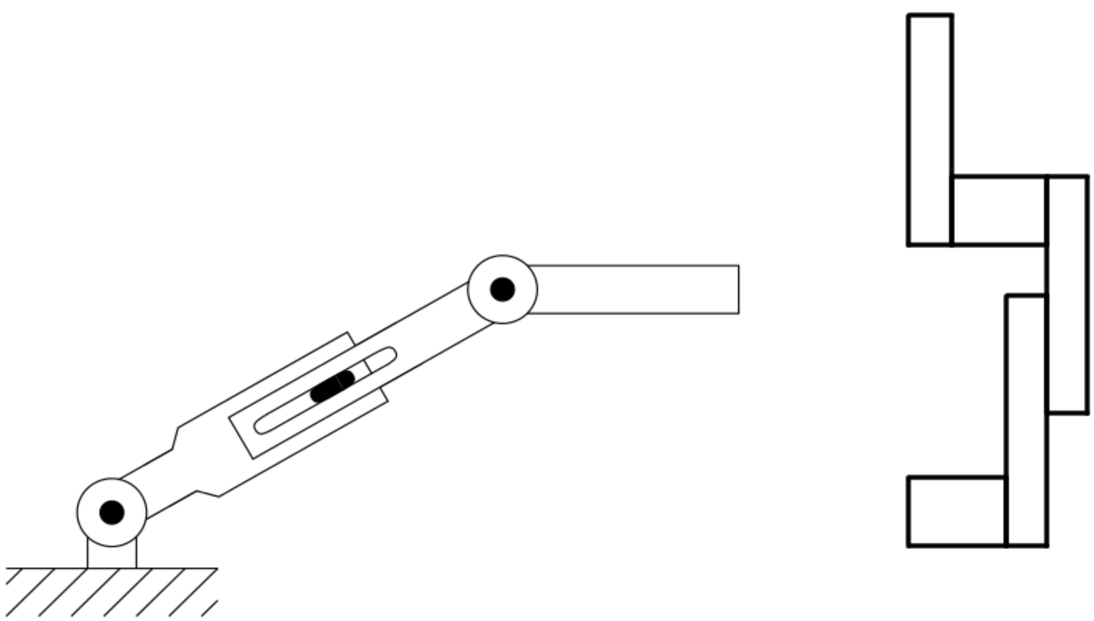
   (d) Consider the robotic arm shown below made up of a prismatic joint (moving vertically) and a revolute joint (positive rotation counter-clockwise in the page). Derive the matrix T_3^0. Assume that the frames shown indicate the X and Y axes of the frames. Do not use the Denavit-Hartenberg convention to obtain a solution; the manipulator is simple enough that you should be able to derive a solution using basic linear algebra.



6. (a) Consider the figure shown below of a PP robot (left: view of the side of the robot, right: view of the front of the robot). Derive the forward kinematics of the robot using the DH-convention; choose your own variables for the missing dimensions.

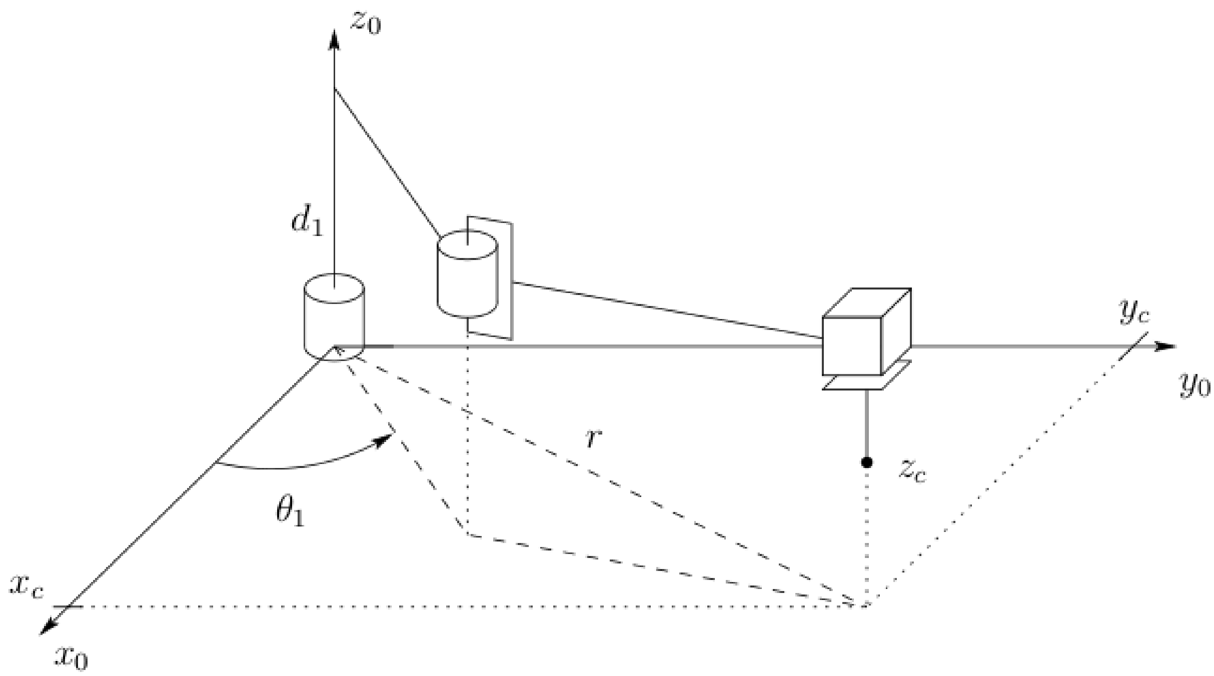
(b) Given the end effector position (say at the end of link 3), solve for the inverse kinematics of the robot.

7. (a) Consider the figure shown below of a RPR robot (left: view of the side of the robot, right: view of the top of the robot). Derive the forward kinematics of the robot using the DH-convention; choose your own variables for the missing dimensions.



(b) Given the end effector position (say at the end of link 4), solve for the inverse kinematics of the robot.

8. (a) Consider the figure shown below of a SCARA robot. Derive the forward kinematics of the robot using the DH-convention; choose your own variables for the missing dimensions.



(b) Given the wrist center location $[x_c\ y_c\ z_c]^T$, solve for the inverse kinematics of the robot.