# CSE4421/5324: Assignment 2 

Burton Ma

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Due: End of class on Mon Feb 25, 2013

1. Consider the RR arm shown in Figure 1.


Figure 1: Left: Front view of arm. Right: Top-down view of arm. In this figure, all joint angles are shown at $0^{\circ}$.
Joints 1 and 2 are connected by a link with a $90^{\circ}$ bend; the bend allows both joints to rotate through $360^{\circ}$ without colliding with each other. The axis of joint 2 is always in the plane $z_{0}=0$. You may assume that the link dimensions $a, b$, and $c$ are always greater than zero, and that $b>c$.
(a) Given the location of frame $\{2\}$ expressed in frame $\{0\}, o_{2}^{0}=(x, y, z)^{T}$, find the values of $\theta_{1}$ and $\theta_{2}$.
(b) A high quality solution to part (a) would check that the point $o_{2}^{0}=(x, y, z)^{T}$ is actually reachable by the robot; how would you check if $o_{2}^{0}$ is reachable? Provide a mathematical expression if possible, although a well written description could also receive full marks.
2. Consider the RR arm shown in Figure 2; this arm is similar to the arm from Question 1 except that the link has a bend of $-45^{\circ}$.


Figure 2: Top-down view of arm.
The axis of joint 2 is always in the plane $z_{0}=0$. You may assume that the link dimensions $a, b$, and $c$ are always greater than zero.

Solve for the forward kinematics of the arm; that is, given joint angles $\theta_{1}$ and $\theta_{2}$, find the orientation and position of frame $\{2\}$ as a $4 \times 4$ homogeneous matrix. If your solution involves a sequence of matrix multiplications then show the sequence of matrices in addition to the overall $4 \times 4$ homogeneous matrix. Use of the Denavit-Hartenberg convention is acceptable, but not required.
3. This question assumes that you have completed Lab 3. In the Matlab simulator for the A150 robot implement the method with signature move (obj, p) that takes as input a goal location p (expressed in the base frame of the robot); the function should then move the origin of frame 5 to the input location, or output a message indicating that the position is not reachable. You should assume that

$$
R_{5}^{0}=\left[\begin{array}{ccc}
-1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{array}\right]
$$

i.e., the gripper is pointing straight down. The path of the end effector should be a straight line. If a straight line path is not possible then the robot should complete as much of the straight line path as is possible; i.e., it should move from its current position towards the goal in a straight line until it can go no further and then stop. Consider adding a method that solves the inverse kinematics problem for the arm, rather than putting all of the inverse kinematics code inside of move; see the final part of this assignment.

Hand in paper copies of Parts 1-2. Submit sim150.m using the command
submit 4421 a2 sim150.m


Figure 3: The wrist of the A150 and A255 robots. $z_{3}$ points out of the page and $y_{5}$ points into the page.

| Joint variable | Range |
| :---: | :---: |
| $\theta_{1}$ | $-175^{\circ}$ to $175^{\circ}$ |
| $\theta_{2}$ | $0^{\circ}$ to $110^{\circ}$ |
| $\theta_{3}$ | $-130^{\circ}$ to $0^{\circ}$ |
| $\theta_{4}$ | $-110^{\circ}$ to $110^{\circ}$ |
| $\theta_{5}$ | $-180^{\circ}$ to $180^{\circ}$ |

Table 1: The joint variable ranges in the Denavit-Hartenberg convention.

