

# Assertions

# Assertions

- Boolean expressions or predicates that evaluate to **true** or **false** in every state
- In a program they express **constraints** on the state **that must be true** at that point
- Associate with
  - » **Individual program statements**
  - » **functions**
  - » **classes**

# Assertions & Correct Programs

## How to write correct programs and know it

– Harlan Mills

- Specify clearly, precisely and succinctly
  - » **What is expected and guaranteed by each component – class, function and statement**
- The essence of documentation
- Essential for debugging
- Aids in fault tolerance

# Assertion Language Symbols

- Arithmetic operators

**+ - \* / ^ (exponent)**

**// div (integer division)**

**\ mod (modulus / remainder)**

- Relational operators

**= ≠ ≤ ≥ < >**

- Boolean operators & logic

**∧ and ∨ or ⊕ xor ¬ ~ not**

**→ implies ↔ iff**

## Assertion Language Symbols – 2

- Semi-strict **and** and **or** – Eiffel only for practical and efficiency reasons
  - » **Also called lazy evaluation in other programming languages**

**and then**

**A and then B**      **Evaluate B only if A is true**

**or else**

**A or else B**      **Evaluate B only if A is false**

## Assertion Language Symbols – 3

- Predicate logic

$\forall$  forall     $\exists$  exists (there exists)

| such that

- it is the case that (it holds that)

- Set operators

$\in$  member\_of     $\notin$  not\_member\_of

$\supset \supseteq \subset \subseteq$  contains

$\not\subset$  does\_not\_contain

$\cap$  intersection     $\cup$  union

$\#S$  number of members of the set S

# Assertion Language Special Symbols

- Special variables related to program semantics

**Result** – result of a function but only in ensure assertions

**Current @** – current object

**Void** – not attached

# Variable before and after values

- Mathematical notation

**name**

value of the variable **name** before its value is changed

**name'**

value of the variable **name** after a its value has changed

**Unlimited context**

- Eiffel notation

**name**

value of the variable **name** after a routine terminates

**old name**

value of the variable **name** before a routine starts

**Limited context**



# Quantified Expression

- Used to express properties about sets of objects

**Quantifier**  $\forall$  forall  $\exists$  exists (there exists)

**Range\_Expr** var\_name : set\_of\_values

**Restriction** Boolean expression or, recursively,  
a quantified expression

**Property** Boolean expression or, recursively,  
a quantified expression

**Quantifier Range\_Expr [ | Restriction ] • Property**

such that // it holds /  
it is the case that //

# Range Expression examples

- Type range – each value is of a given type

**v : VEHICLE**

- Sequence range – each value is in a sequence

**k : low .. high**

- Member range – each value is a member in a set

**c ∈ children**

# Textual Notation example

**class CITIZEN feature**

**name, sex, age : VALUE**

**spouse : CITIZEN**

**children, parents : SET[CITIZEN]**

**single : BOOLEAN ensure Result iff ( spouse = Void ) end**

**divorce**

**require not single**

**ensure single and ( old spouse ) . single**

**end**

**invariant**

**single or spouse.spouse = Current**

**parents.count = 2**

**for\_all c member\_of children it\_holds**

**( exists p member\_of c.parents it\_holds p = Current )**

**end**

# Mathematical Notation example

**class CITIZEN feature**

**name, sex, age : VALUE**

**spouse : CITIZEN**

**children, parents : SET[CITIZEN]**

**single : BOOLEAN ensure Result  $\leftrightarrow$  ( spouse = Void ) end**

**divorce**

**require  $\sim$  single**

**ensure single  $\wedge$  spouse . Single**

**end**

**invariant**

**single  $\vee$  spouse . spouse = @**

**parents . count = 2**

**$\forall c \in \text{children} \cdot (\exists p \in c . \text{parents} \cdot p = @)$**

**}**

## Specifying Members of a Set

- Set enumeration – list the members

$$S = \{ a, e, i, o, u \}$$

The set of vowels in the English alphabet

- Set comprehension – logically specify members  
Notice that the forall is implicit not explicit

$$\{ x, y : \text{Integer} \mid (0 < x < 10) \wedge (1 \leq y \leq 9) \cdot x^3 + y^3 \}$$

The set of the sums of pairs of the cubes of single digit integers greater than zero

# Pre-Conditions

- Statement syntax
  - » **require boolean expression**
- Where within function/procedure
  - » **write just before the local clause, if it exists**

```
nonZero ( row , col : INTEGER ) : BOOLEAN  
-- Result true if non-zero element at <row, col>  
require 0 < row and row < MaximumRow + 1  
         0 < col and col < MaximumCol + 1  
  
do  
  
...  
  
end
```

# Post-Conditions

- Statement syntax
  - » **ensure** **boolean expression**
- Where within function/procedure
  - » **write just before the end of body**

```
NonZero ( row , col : INTEGER ) : BOOLEAN  
-- Result true if non-zero element at <row, col>  
do  
...  
ensure Result =  
  ( search_by_row(row, col) /= void and  
    search_by_row(row, col).data /= 0 )  
end
```

## State changes

- Show relationship between initial and final values
- At the end of the body the final values are in effect
- Refer to initial values using the keyword **old**

```
addElement ( element : TYPE )  
require size < Capacity  
do  
  
...  
  
ensure size = old size + 1  
end
```



# Assertions are tagged

- Tag names are used to identify assertions

```
addElement ( element : TYPE )  
require enough_space: size < Capacity  
do  
  
...  
  
ensure one_larger: size = old size + 1  
end
```

## Non-executable assertions

- Use **comments** if you cannot write an executable assertion
- Use already defined functions or custom written functions

```
insert_in_row(matElem : MATRIX_ELEMENT)  
  -- Insert the matrix element in the current row "row"  
  require ...  
  local ...  
  do ...  
  ensure  
    -- contains(MatrixElement(data, row, column)) at < row, column >  
  end
```

# Loop Invariants & Loop Syntax

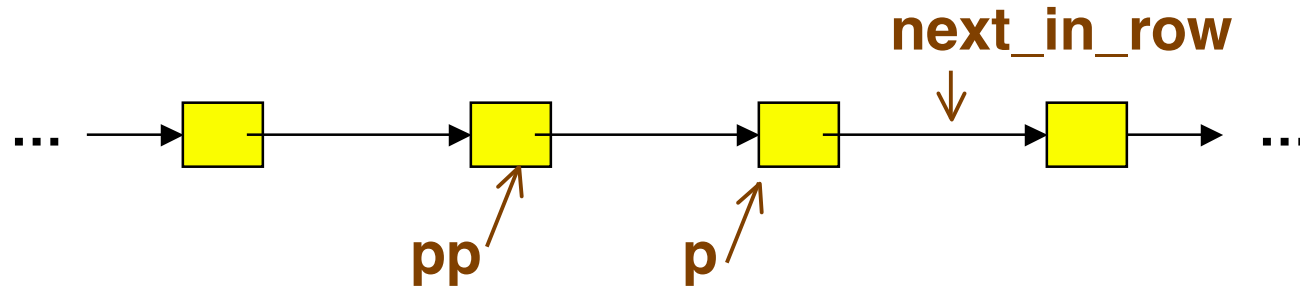
**from**  
    **init statements**  
**invariant**  
    **assertions for invariant**  
**until**  
    **exit condition**  
**loop**  
    **body statements**  
**variant**  
    **integer expression**  
**end**

- Can invoke Boolean functions
- Use agents to implement predicate calculus expressions

- Always non negative
- Body decreases value on every iteration
- Ideally 0 on loop exit

# Loop Invariant Example

- Inserting an element into a sorted singly linked list



```
row := matElem.row ; column := matElem.column  
from p := rowList @ row  
invariant ???  
until  
  p = void or p.column >= column  
loop  
  pp := p ; p := p.next_in_row  
end
```

## Loop Invariant Example – 2

- Using mathematical notation

**invariant**

**predecessor\_relation:**  $(pp = \text{void} \wedge p = \text{head})$

$\forall (pp \neq \text{void} \wedge pp . \text{next} = p)$

**predecessor\_before\_data:**  $pp \neq \text{void} \wedge pp . \text{data} < \text{data}$

**data\_less\_than:**  $\forall k : \text{head} .. pp \bullet k . \text{data} < \text{data}$

## Loop Invariant Example – 3

- Eiffel executable assertion.
- Column\_less\_than uses an agent to implement the invariant
  - > **Agents and loop invariants are discussed in other slides**

```
from p := rowList @ row
invariant
  predecessor_relation : (pp = void and p = rowList @ row)
                        or (pp /= void and pp.next_in_row = p)
  predecessor_before_column: pp = void or pp.column < column
-- forall k : rowList @ row .. pp :: k.column < column
  data_less_than : column_limit( rowList @ row, pp,
                                agent less_than(?, column) )
end
```

# Check Assertion

- Within the body of a routine you can insert a **check** clause
- The **check** clause is executed and if an assertion is false then an exception occurs
- Used to remind the reader of a non obvious fact that could be deduced

```
If full then error := overflow  
else  
  check  
    representation_exists : representation /= Void  
  end  
  representation.put(x) ; error := none  
end
```

# Class Invariants

- Appear in the **invariant** clause just before the end of the class definition

```
class SPARSE_MATRIX  
...  
invariant  
  actualRows <= maxRowCol  
  actualCols <= maxRowCol  
  -- forall row : maxNonzeroRow + 1 .. actualRows  
  --           :: empty ( rowList [ row ] )  
  -- forall col : maxNonzeroCol + 1 .. actualCols  
  --           :: empty ( colList [ col ] )  
end -- SPARSE_MATRIX
```



## Class Invariants – 2

- Class invariants define which states of the ADT are valid
- True at stable times
  - » **After make (object creation)**
  - » **After every exported feature call**
    - > **Could be false during a feature call as various sub-states change**
- Invariant is implicitly a part of every pre and post condition

# Class Invariants – examples

- See slides 9 & 10 in this set of slides
  - » **Relationship between parents and children**
  - » **Relationship between spouses**
  
- See Abstract data type documentation slides 18 .. 23
  - » **Relationship between first and last pointers in a circular queue and the length of the queue**

# General Guideline

- Assertions may be written in many ways
  - » **Select the representation to be as clear and easy to understand as possible**
    - > **Point is to convey information, not provide a puzzle to be solved**
  - » **Use notation that is close to the meaning of the relationships involved – no need to restrict to first order predicate calculus**
    - > **Set notation**
    - > **Bag notation**
    - > **Sequence notation**

# Assertion Monitoring

- Eiffel provides multiples levels of assertion monitoring
  - » **See the project settings & page 393**
- Always should be on during debugging
- Turn off as little as possible only if time is critical and the system can be trusted