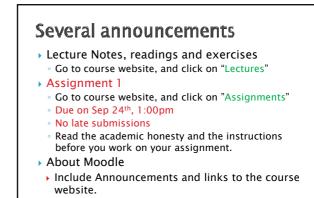
Math/CSE 1019C: Discrete Mathematics for Computer Science Fall 2012

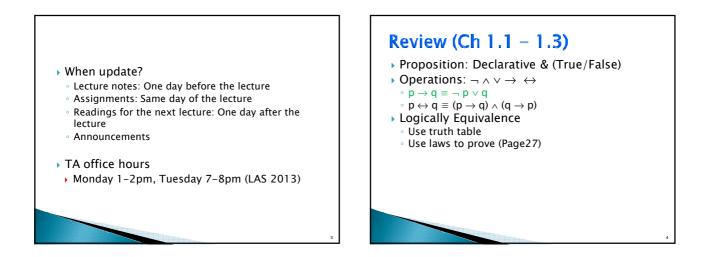
Jessie Zhao

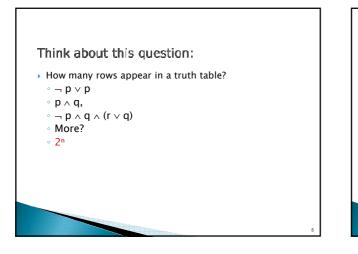
jessie@cse.yorku.ca

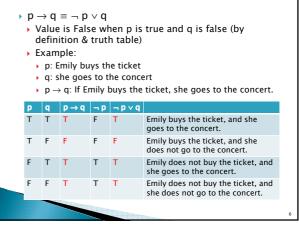
Course page: http://www.cse.yorku.ca/course/1019

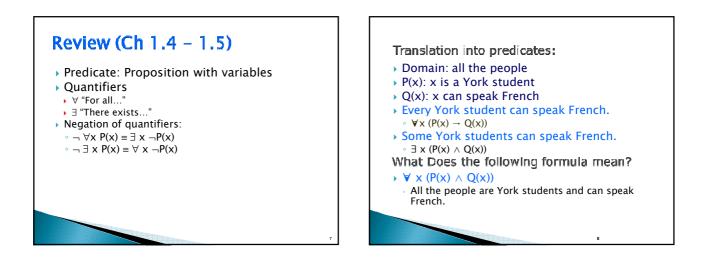


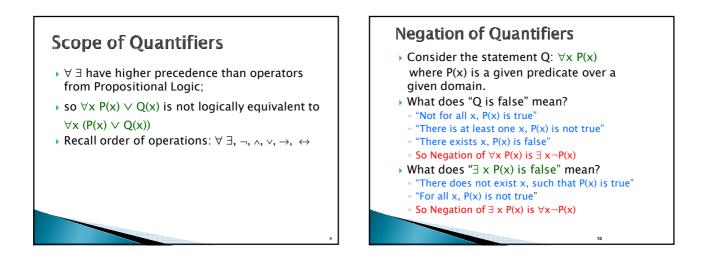
Course website is the best place to check.

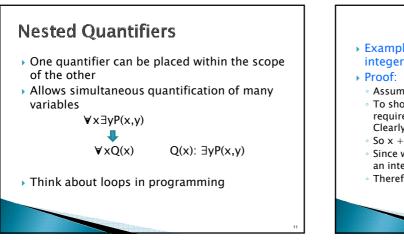


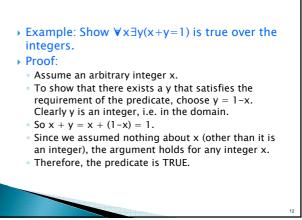






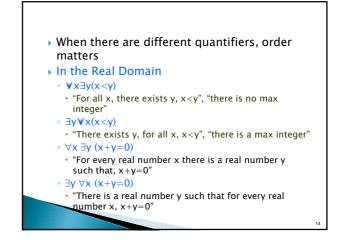






Order of Nested Quantifiers

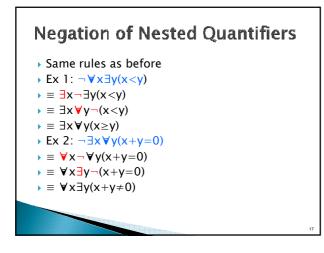
- When there are only one kind of quantifiers (universal or existential) in a statement, then the change of order does not change the meaning of the statement:
 - $\forall x \forall y P(x,y) \equiv \forall y \forall x P(x,y)$
 - $\exists x \exists y P(x,y) \equiv \exists y \exists x P(x,y)$

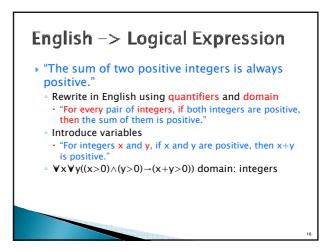


	When true?	When false?
$\forall x \forall y P(x,y)$ $\forall y \forall x P(x,y)$	P(x,y) is true for every pair (x,y)	A pair (x,y) exists for which P(x,y) is false
∀x∃yP(x,y)	For every x, there is a y for which P(x,y) is true	There is an x such that P(x,y) is false for every y
∃x∀yP(x,y)	There is an x for which P(x,y) is true for every y	For every x, there is a y for which P(x,y) is false
∃x∃yP(x,y) ∃y∃xP(x,y)	There is a pair (x,y) for which P(x,y) is true	P(x,y) is false for all pairs (x,y)

Examples

- > What is the truth value of the following:
 - ∘ $\exists x \forall y(x+y=0)$ domain: integers
 - ∘ $\exists x \forall y(xy=0)$ domain: integers
 - $\forall x \neq 0 \exists y(y=1/x)$ domain: real numbers
 - ∘ $\forall x \forall y \exists z(z=(x+y)/2)$ domain: integers

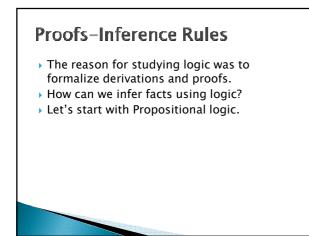


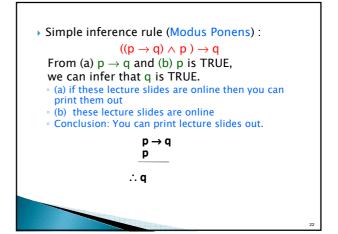


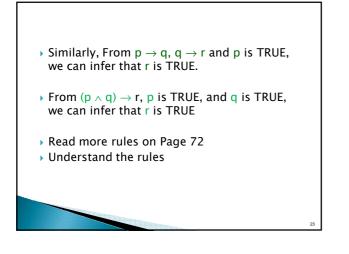
Logical Expression -> English ∃x∀y∀z(friends(x,y)∧friends(x,z)∧(y≠z)→¬friends(y,z)) Domain of x, y and z: all students "There is a student x such that for all students y and all students z, if x and y are friends, x and z are friend and z and y are not the same student, then y and z are not friend." "There is a student none of whose friends are also friends with each other."

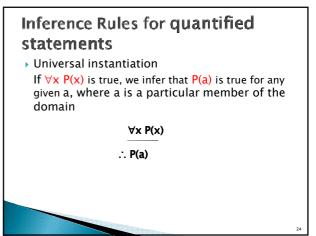
Readings and notes

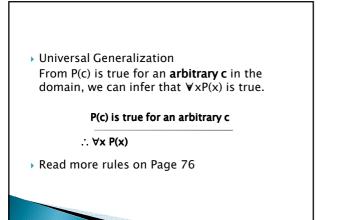
- Read Ch1.1-1.5
- Practice translating English sentences to propositions and predicates
- Practice to use truth tables
 Practice proving logical equivalence by manipulating compound propositions
- Understand the difference and relationship between propositions, predicates and quantifications.

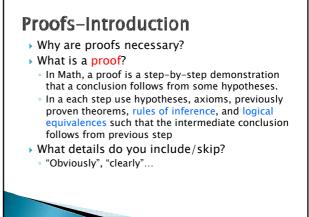












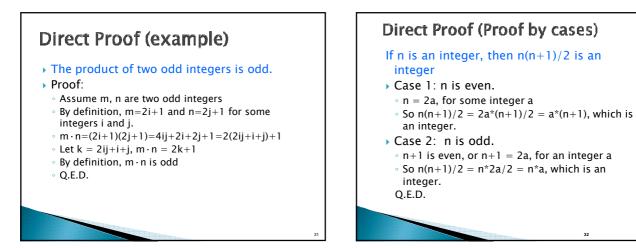
Function of Constant SeriesStatement that can be proved to be true Axiom: A statement which is given to be true Lemma: A 'pre-theorem' that is needed to prove a theorem Corollary: A 'post-theorem' that follows from a theorem Corollary: A 'post-theorem' that follows from a theorem

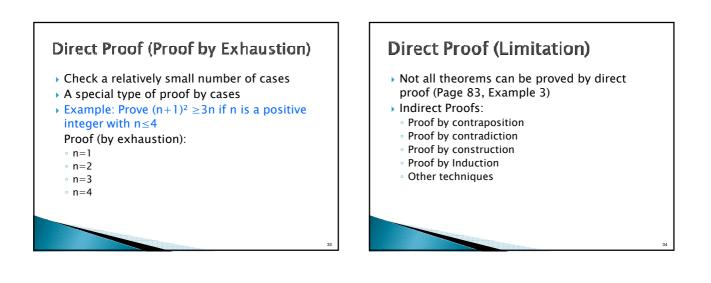
Types of Proofs

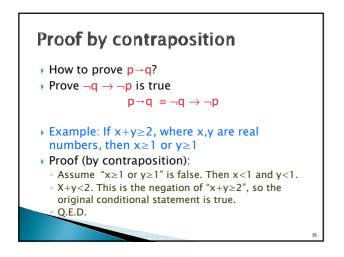
- > Direct proofs (including Proof by cases)
- Proof by contraposition
- Proof by contradiction
- Proof by construction
- Proof by Induction
- Other techniques

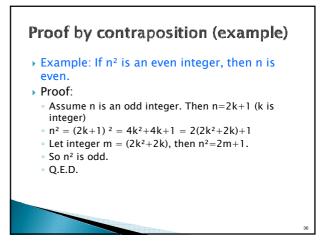
Direct Proof

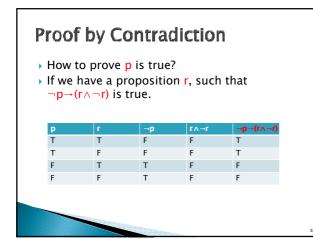
- Leads from hypothesis to the conclusion
- How to prove $p \rightarrow q$?
- Assume p is true
- Use rules of inference, axioms, lemmas, definitions, proven theorems, ...
- Conclude that q must be true
- > Q.E.D. (used to signal the end of a proof)

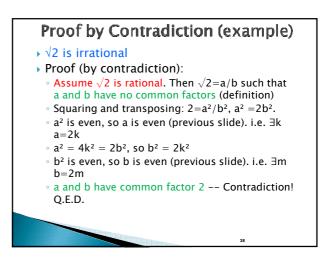


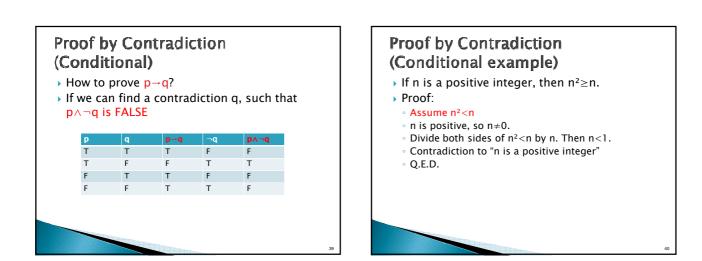


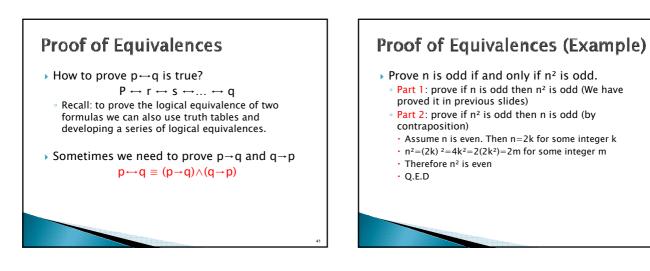


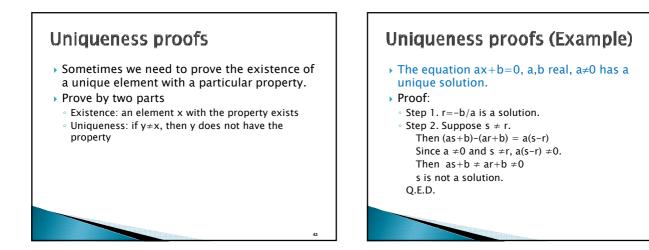












Disproof by counterexample

- One way to prove p is not true
- Find a counterexample such that p is false
- Show the following is FALSE: If x, y are irrational, x + y is irrational.
 Proof: x= √2, y= √2 are irrational, and x+y=0 is
 - Proof: $x = \sqrt{2}$, $y = -\sqrt{2}$ are irrational, and x+y=0 is rational.

Readings and notes

- Read Ch1.5-1.8
- > Understand the order and scope of the quantification
- Practice translating between English and logical expressions
- Understand the proof methods
- Practice proof a lot!
- Recommended book: "How to read and do proofs" by Daniel Solow

Examples

- Show that if n is an odd integer, there is a unique integer k such that n is the sum of k-2 and k+3.
- Prove that there are no solutions in positive integers x and y to the equation $2x^2 + 5y^2 = 14$.
- If x³ is irrational then x is irrational