Math/CSE 1019C: Discrete Mathematics for Computer Science
Fall 2012

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Course page:
http://www.cse.yorku.ca/course/1019

Class Time and Office Hours

- Class time:
  - Monday: 19:00pm – 22:00pm
- Class location: SLH A
- Office hours
  - Monday 2:00pm – 4:00pm or by appointment (TEL 3056)
- Contact me by Email
  - Use a York account
  - Start your subject line with "[1019]"
  - Sign with your full name
  - Send messages in plain text

- TAs
  - Arindam Das arindam@cse.yorku.ca
  - Wendy Ashlock washlock@cse.yorku.ca
  - Maria Angel Marquez Andrade cse01009@cse.yorku.ca
- TA office hours? Choose Two from the following
  - (1) Mon: 1-2 pm
  - (2) Tue: 1-2 pm
  - (3) Tue: 7-8 pm
  - (4) Tue: 8-9 pm
  - (5) Wed: 7-8 pm
  - (6) Wed: 8-9 pm
  - (7) Thu: 8-9 pm

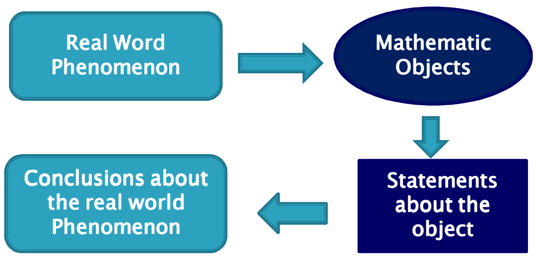
Evaluation and Grading

- 7 assignments (15%)
- 3 Tests (45%)
  - Oct. 15st, Nov 5th, Nov 26th (tentative)
  - No deferred tests
- Final Exam (40%) Dec. ?

Assignments Policy

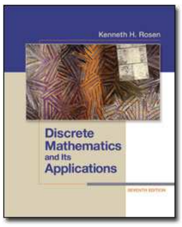
- Academic Honesty.
  - Solutions you hand in for homework assignments must be your own work.
  - Visit the class webpage for more details on academic policy.
- Use the Dropbox to submit your assignments.
  - Locates in the 1st floor of LAS (previously known as CSE)
- Assignments submitted late will not be graded. The solutions will be posted when the deadline is reached.
  - Missed assignments
    - No reason: graded with 0
    - With reason: transferred to final exam value

Why study Mathematic?



Textbook

Kenneth H. Rosen.
Discrete Mathematics
and Its Applications,
7th Edition.
McGraw Hill,
2012.



Course objectives

We will focus on two major goals:

- Basic tools and techniques in discrete mathematics
  - Propositional logic
  - Set Theory
  - Simple algorithms
  - Induction, recursion
  - Counting techniques
- Precise and rigorous mathematical reasoning
  - Writing proofs

List of Topics in the Textbook

- Ch 1: Logic and Proofs.
- Ch 2: Sets, functions, sequences, sums.
- Ch 3: Algorithms.
- Ch 5: Induction and recursion.
- Ch 6: Counting Techniques.
- Ch 8 Advanced counting techniques.

To do well you should:

- Study with pen and paper
- Ask for help immediately
- Practice, practice, practice…
- Follow along in class rather than take notes
- Ask questions in class
- Read the book, not just the slides

Propositional Logic

- A formal mathematical “language” for precise reasoning
  - Truth values, truth tables
  - Boolean logic: $\vee$ $\wedge$ $\neg$
  - Implications: $\rightarrow$ $\leftrightarrow$
- All of these are based on ideas we use daily to reason about things.

Propositions

- Declarative sentence
- Must be either True or False.

Propositions:

- CNN Tower is in Toronto
- Toronto is the capital of Canada.
- 1+1=2

Not propositions:

- There are x students in this class.
  - Neither true or false
- Do you like this course?
  - Not declarative

Propositions can be represented by variables:

- P: CNN Tower is in Toronto
- q: Toronto is the capital of Canada.
- r: 1+1=2

Truth value: True or False (T or F)

- p: T
- q: F
- r: T

Negation

- ¬p ("not p")
  - "It is not the case that p"
  - p: Today is Monday
  - ¬ p: Today is not Monday
- Truth tables

p	¬p
T	F
F	T

Conjunction

- Conjunction: p ∧ q ("p and q")
  - p: It is blew freezing.
  - q: It is snowing
  - p ∧ q : It is blew freezing and snowing.
- p ∧ q is true if and only if both p and q are true and false otherwise

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

- p ∨ q ("p or q")
  - p: A student taking 1019 is from CSE Department
  - q: A student taking 1019 is form Math Department
  - p ∨ q: A student taking 1019 is from CSE Department or Math Department.
- p ∨ q is false when both p and q are false and is true otherwise

p	q	p ∨ q
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive OR (XOR)

- p ⊕ q ("p or q, but not both")
  - In a steak house, you can either choose a salad or a soup, but not both
- p ⊕ q is true if p and q have different truth values and is false otherwise

p	q	p ⊕ q
T	T	F
T	F	T
F	T	T
F	F	F

Conditional

- Conditional p → q ("if p then q")
- p: *hypothesis*, q: *conclusion*
  - If you turn in a homework late, it will not be graded
- p → q is false when p is true and q is false, and true otherwise.

p	q	p → q	¬ p ∨ q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Logical Equivalence

- $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.
- $p \rightarrow q \equiv \neg p \vee q$
- Truth tables are the simplest way to prove such facts.
- We will learn other ways later.

Contrapositive

- Contrapositive of $p \rightarrow q$ is $\neg q \rightarrow \neg p$
- Any conditional and its contrapositive are logically equivalent (have the same truth table) – Check by writing down the truth table.
  - If you turn in a homework late, it will not be graded.
  - If your homework is graded, you do not turn in the home work late.

Converse

- Converse of $p \rightarrow q$ is $q \rightarrow p$
- Not logically equivalent to conditional
  - If you won the lottery, you are rich.

Inverse:

- Inverse of $p \rightarrow q$ is $\neg p \rightarrow \neg q$

- Compare using Truth table: Conditional, Contrapositive, Converse, and Inverse

p	q	$p \rightarrow q$	$\neg q \rightarrow \neg p$	$q \rightarrow p$	$\neg p \rightarrow \neg q$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	F	T	T	T	T

Biconditional

- $p \leftrightarrow q$("p if and only if q", "iff")
- True if p,q have same truth values, false otherwise. Q: How is this related to XOR?
- Can also be defined as $(p \rightarrow q) \wedge (q \rightarrow p)$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Compound Propositions

- Formed from existing propositions using logic operators
- Example: $p \wedge q \vee r$: Could be interpreted as $(p \wedge q) \vee r$ or $p \wedge (q \vee r)$
- precedence order: $\neg \wedge \vee \rightarrow \leftrightarrow$
- (Overruled by parentheses)
- We use this order to compute truth values of compound propositions.

Tautology

- A compound proposition that is always TRUE.
  - $q \vee \neg q$
  - $p \vee T$
- Logical equivalence redefined: p,q are logical equivalences if $p \leftrightarrow q$ is a tautology.

$$p \equiv q$$

- "⇔" is sometimes used instead of "≡"
- Intuition: $p \leftrightarrow q$ is true precisely when p,q have the same truth values.
- We will learn other ways later

Manipulating Propositions

- Compound propositions can be simplified by using simple rules.
- Read page 25 – 28.
- Some are obvious, e.g. Identity, Domination, Idempotence, double negation, commutativity, associativity
- Less obvious: Distributive, De Morgan's laws, Absorption

Distributive Laws

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

Intuition (not a proof!) – For the LHS to be true: p must be true and q or r must be true. This is the same as saying p and q must be true or p and r must be true.

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Intuition (less obvious) – For the LHS to be true: p must be true or both q and r must be true. This is the same as saying p or q must be true and p or r must be true.

Proof: use truth tables.

De Morgan's Laws

$\neg(q \vee r) \equiv \neg q \wedge \neg r$

Intuition – For the LHS to be true: neither q nor r can be true. This is the same as saying q and r must be false.

$\neg(q \wedge r) \equiv \neg q \vee \neg r$

Intuition – For the LHS to be true: $q \wedge r$ must be false. This is the same as saying q or r must be false.

Proof: use truth tables.

Using the laws

- Q: Is $(p \wedge q) \rightarrow (p \rightarrow q)$ a tautology?
- Can use truth tables
- Can write a compound proposition and simplify

- Limitations of Propositional Logic
  - Refer to (Constant) objects
  - How about "x>0"?
- A more general language: Predicate logic

Predicate Logic

- A predicate is a proposition that is a function of one or more variables.
  - P(x): x is larger than 0.
  - P(1) is true, P(-2) is false,....
- "x": variable
- P(x): the value of the propositional function P at x
- Multiple variables
  - P(x,y): x + y = 5
  - P(2,3) is true, P(4,0) is false,....

Quantifiers

- Describes the values of a variable that make the predicate true.
- Determines the truth value of the predicate
- Domain or universe: a property is true for all values in a particular domain.
- Two Popular Quantifiers
  - Universal
  - Existential

Universal Quantifier

- Universal: ∀x P(x) – "P(x) is true for all x in the domain"
  - Also called: "for all… ", "for every… ", "for each…", "all of …",…
  - ∀x P(x) is true when P(x) is true for every x in the domain
  - ∀x P(x) is false when P(x) is not always true when x is in the domain (there exists a value of x that P(x) is false: use counterexample)
  - Domain: real numbers
    - $(\forall x>2)(x^2>4)$
    - $(\forall x>0)(x^2>1)$

Existential Quantifier

- Existential: ∃x P(x) – "P(x) is true for some x in the domain"
  - Also called "There exists…", "There is…", "For some…", "For at least one…"
  - ∃x P(x) is true when there is an x in the domain for which P(x) is true.
  - ∃x P(x) is false when P(x) is false for every x in the domain.
  - Domain: real numbers
    - $\exists x\ (x>1)$
    - $(\exists\ x>1)\ (x=x+1)$

Scope of Quantifiers

- ∀ ∃ have higher precedence than operators from Propositional Logic; so $\forall x\ P(x) \lor Q(x)$ is not logically equivalent to $\forall x\ (P(x) \lor Q(x))$
- Logical Equivalence: P ≡ Q iff they have same truth value no matter which domain is used and no matter which predicates are assigned to predicate variables.

Negation of Quantifiers

- $\neg\ \forall x\ P(x) \equiv \exists\ x\ \neg P(x)$
- $\neg\ \exists\ x\ P(x) \equiv \forall\ x\ \neg P(x)$
  - "There is no student who can answer this question."
  - "All Americans eat cheeseburger."
- Careful: The negation of "All Americans eat cheeseburger." is NOT "No Americans eat cheeseburger"!

Nested Quantifiers

- Allows simultaneous quantification of many variables.
- E.g. – domain integers, $\exists x \exists y \exists z \; x^2 + y^2 = z^2$
- Domain real numbers: $\forall x \forall y \exists z \, (x < z < y) \vee (y < z < x)$

Nested Quantifiers

- The order of quantifiers
- In the real domain:
  - $\forall x \exists y \, (x+y=0)$: "For every real number x there is a real number y such that, x+y=0"
  - $\exists y \forall x \, (x+y=0)$: "There is a real number y such that for every real number x, x+y=0"

Readings and notes

- Read Ch1.1-1.5
- Practice translating English sentences to propositions and predicates
- Practice to use truth tables
- Practice proving logical equivalence by manipulating compound propositions
- Understand the difference and relationship between propositions, predicates and quantifications.
- Recommended Exercises are listed on the website

- Part of the course contents come from Prof. Eric Ruppert, Prof. Suprakash Datta, and Jing Yang, who have taught this course previously.