Math/CSE 1019C: Discrete Mathematics for Computer Science

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Course page:
http://www.cse.yorku.ca/course/1019

Permutation

- For any integer n>0, the number of permutations of a set with n elements is n!
  - A permutation of a set of elements is an ordering of the elements.
  - E.g. the set of elements {a, b, c} can be ordered in the following ways:
    - abc acb cba bac bca cab
  - By the product rule, there are n(n-1)(n-2)..1=n! permutations

- Ex: Suppose there are 50 students in the class,
  - In how many ways can the whole class stand in a line?

    50!
  - In how many ways can we select three students to stand in a line?

    50*49*48

r-permutation

- An r-permutation is an ordering of r elements of a set of n elements, denoted by P(n,r)
  - E.g. the 2-permutations of the set of elements {a, b, c} are:
    - ab ac ba bc ca cb
  - By the product rule, there are n(n-1)(n-2)....(n-r+1) r-permutations

r-permutation

- P(n,r) = n!/(n-r)! = n(n-1)(n-2)....(n-r+1) for 0≤r≤n
- Special cases:
  - P(n,0) = 1
  - P(0,0) = 1
  - P(n,n) = n!

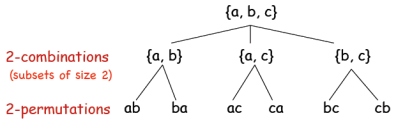
- Recall: How many one-to-one functions are there from a set with m elements to one with n elements?
  - n(n-1)...(n-m+1) when m≤n
  - 0 when m>n

- For the solitaire hand that show initially
  - How many possible hands?
    p(52,7)
  - How many possible hands with no Aces?
    p(48,7)
  - How many possible hands with one or more Aces?
    P(52,7)-P(48,7)

Combinations

- An r-combination is an ***unordered*** selection of r elements of a set of n elements, denoted by C(n,r)
- E.g. the 2-combinations of the set of elements {a,b, c} are:
  - {a,b} {a,c} {b,c}

Combinations & Permutations



- There are r! permutation of each subset
- There are more r-permutation than r-combinations.

Combinations

$$C(n,r) = \frac{P(n,r)}{P(r,r)} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

for $0 \le r \le n$

Corollary: C(n,r) = C(n,n-r)

- For a deck of 52 cards,
- How many poker hands of five cards can there be?
  C(52,5)=2,598,960
- How many ways are there to select 47 cards?
  C(52,47)=C(52,5)=2,598,960