CS345 Notes for Lecture 10/14–16/96

Saraiya's Containment Test

- Containment of CQ's is NP-complete in general.
- Sariaya's algorithm is a polynomial-time test of $Q_1 \subseteq Q_2$ for the common case that no predicate appears more than twice among the subgoals of Q_1.
  - □ They can appear any number of times in Q_2.
- The algorithm is a reduction to 2SAT and yields a linear-time algorithm.
- Our algorithm is more direct, but quadratic.

The idea is to pick a subgoal of Q_2, and consider the consequences of mapping it to the two possible subgoals of Q_1. Follow all consequences of this choice: subgoals that must map to subgoals, and variables that must map to variables.

- If we know $p(X_1, \ldots, X_n)$ must map to $p(Y_1, \ldots, Y_n)$, then infer that each X_i must map to Y_i.
- If $p(X_1, \ldots, X_n)$ is a subgoal of Q_2, and we know X_i maps to some variable Z, and only one of the p-subgoals of Q_1 has Z in the ith component (or Q_1 only *has* one p-subgoal), then conclude $p(X_1, \ldots, X_n)$ maps to this subgoal.

One of two things must happen:

1. We derive a contradiction: a subgoal or variable that must map to two different things. If so, try the other choice if there is one; fail if there is no other choice.
2. We close the set of inferences we must make. Then we can forever forget about the question of how to map the determined subgoals and variables. We have found one mapping that works and that can't interfere with the

mapping of any other subgoals or variables, so we make another arbitrary choice if there are any unmapped subgoals.

Example: Let us test $C_2 \subseteq C_1$, where:

C_1: `p(X) :- a(X,Y) & b(Y,Z) & b(Z,W) & a(W,X)`
C_2: `p(A) :- a(A,B) & a(B,A) & b(A,C) & b(C,B)`

- Note this simple example omits some options: C_2 could have a predicate appearing only once in the body, and C_2 could have 3 or more occurrences of some predicates.

Here is a description of inferences that might be made:

(1) Suppose $a(X,Y) \to a(A,B)$
(2) Then $X \to A$, $Y \to B$
(3) Now, $b(Y,Z) \to b(B,?)$
(4) Since there is no $b(B,?)$, fail
(5) Thus, we must map $a(X,Y) \to a(B,A)$
(6) Then $X \to B$ and $Y \to A$,
(7) $b(Y,Z) \to b(A,C)$, $Z \to C$,
(8) $b(Z,W) \to b(C,B)$, $W \to B$
(9) Now, $a(W,X)$ must map to $a(B,B)$
(10) Since $a(B,B)$ does not exist, fail

- Note, however, that if the last subgoal of C_2 were $b(C,A)$, we would have $W \to A$ at line (8) and $a(W,X) \to a(A,B)$ at line (9).

  - □ That completes the containment mapping successfully, with $X \to B$, $Y \to A$, $Z \to C$, and $W \to A$.