importance resampling technique, which is part of the particle filter, goes back to two seminal papers by Rubin (1988) and Smith and Gelfand (1992). Stratified sampling was first invented by Neyman (1934). In the past few years, particle filters have been studied extensively in the field of Bayesian statistics (Doucet 1998; Kitagawa 1996; Liu and Chen 1998; Pitt and Shephard 1999). In AI, particle filters were reinvented under the name *survival of the fittest* (Kanazawa et al. 1995); in computer vision, an algorithm called *condensation* by Isard and Blake (1998) applies them to tracking problems. A good contemporary text on particle filters is due to Doucet et al. (2001).

4.6 Exercises

1. In this exercise, you will be asked to implement a histogram filter for a linear dynamical system studied in the previous chapter.

   (a) Implement a histogram filter for the dynamical system described in Exercise 1 of the previous chapter (see page 81). Use the filter to predict a sequence of posterior distributions for $t = 1, 2, \ldots, 5$. For each value of t, plot the joint posterior over x and $\dot{x}$ into a diagram, where x is the horizontal and $\dot{x}$ is the vertical axis.

   (b) Now implement the measurement update step into your histogram filter, as described in Exercise 2 of the previous chapter (page 82). Suppose at time $t = 5$, we observe a measurement $z = 5$. State and plot the posterior before and after updating the histogram filter.

2. You are now asked to implement the histogram filter for the nonlinear studied in Exercise 4 in the previous chapter (page 83). There, we studied a nonlinear system defined over three state variables, and with the deterministic state transition

$$\begin{pmatrix} x' \\ y' \\ \theta' \end{pmatrix} = \begin{pmatrix} x + \cos\theta \\ y + \sin\theta \\ \theta \end{pmatrix}$$

   The initial state estimate was as follows:

$$\mu = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \begin{pmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 10000 \end{pmatrix}$$

   (a) Propose a suitable initial estimate for a histogram filter, which reflects the state of knowledge in the Gaussian prior.

(b) Implement a histogram filter and run its prediction step. Compare the resulting posterior with the one from the EKF and from your intuitive analysis. What can you learn about the resolution of the x-y coordinates and the orientation θ in your histogram filter?

(c) Now incorporate a measurement into your estimate. As before, the measurement shall be a noisy projection of the x-coordinate of the robot, with covariance $Q = 0.01$. Implement the step, compute the result, plot it, and compare it with the result of the EKF and your intuitive drawing.

Notice: When plotting the result of a histogram filter, you can show multiple density plots, one for each discrete slice in the space of all θ-values.

3. We talked about the effect of using a single particle. What is the effect of using $M = 2$ particles in particle filtering? Can you give an example where the posterior will be biased? If so, by what amount?

4. Implement Exercise 1 using particle filters instead of histograms, and plot and discuss the results.

5. Implement Exercise 2 using particle filters instead of histograms, and plot and discuss the results. Investigate the effect of varying numbers of particles on the result.