4. Now consider a simple kinematic model of an idealized *bicycle*. Both tires are of diameter d, and are mounted to a frame of length l. The front tire can swivel around a vertical axis, and its steering angle will be denoted α. The rear tire is always parallel to the bicycle frame and cannot swivel.

   For the sake of this exercise, the pose of the bicycle shall be defined through three variables: the x-y location of the center of the front tire, and the angular orientation θ (yaw) of the bicycle frame relative to an external coordinate frame. The controls are the forward velocity v of the bicycle, and the steering angle α, which we will assume to be constant during each prediction cycle.

   Provide the mathematical prediction model for a time interval Δt, assuming that it is subject to Gaussian noise in the steering angle α and the forward velocity v. The model will have to predict the posterior of the bicycle state after Δt time, starting from a known state. If you cannot find an exact model, approximate it, and explain your approximations.

5. Consider the kinematic bicycle model from Exercise 4. Implement a sampling function for posterior poses of the bicycles under the same noise assumptions.

   For your simulation, you might assume $l = 100cm$, $d = 80cm$, $\Delta t = 1sec$, $|\alpha| \leq 80°$, $v \in [0; 100]cm/sec$. Assume further that the variance of the steering angle is $\sigma_\alpha^2 = 25°^2$ and the variance of the velocity is $\sigma_v^2 = 50cm^2/sec^2 \cdot v^2$. Notice that the variance of the velocity depends on the commanded velocity.

   For a bicycle starting at the origin, plot the resulting sample sets for the following values of the control parameters:

problem number	α	v
1	25°	$20cm/sec$
2	−25°	$20cm/sec$
3	25°	$90cm/sec$
4	80°	$10cm/sec$
1	85°	$90cm/sec$

   All your plots should show coordinate axes with units.

6. Consider once again the kinematic bicycle model from Exercise 4. Given an initial state x, y, θ and a final x' and y' (but no final θ'), provide a mathematical formula for determining the most likely values of α, v, and θ'. If