Homework Assignment #3
Due: June 4, 2012 at 7:00 p.m.

1. If L_1 and L_2 are two languages over the alphabet Σ, then we define $L_1 \diamond L_2$ to be the language $\{x_1y_1x_2y_2\ldots x_ny_n : n \geq 0,$ each x_i and y_i are in $\Sigma^*, x_1x_2\ldots x_n \in L_1,$ and $y_1y_2\ldots y_n \in L_2\}$. Each string in this language is formed by interleaving one string from L_1 with one string from L_2.

(a) If $L_1 = \{\text{BOB, MARY}\}$ and $L_2 = \{\text{PAUL}\}$, write down three strings that are in $L_1 \diamond L_2$.

(b) If $L_1 = \{\text{a}\}^*$ and $L_2 = \{\text{aab}\}$, give a precise description of all the strings that are in $L_1 \diamond L_2$.

(c) Prove that, for all regular languages L_1 and L_2, $L_1 \diamond L_2$ is also regular.

The level of detail in your answer should be similar to the level of detail given in the proof of Theorem 1.47 in the textbook. If you construct an automaton for $L_1 \diamond L_2$, you should give a precise description of which strings take your automaton to each state (although you do not have to give a formal proof of this claim).