Homework Assignment #9
Due: March 30, 2011 at 2:30 p.m.

1. **(a)** Let $L = \{\langle M \rangle : M$ is a Turing machine that halts on input $\varepsilon\}$. Show that L is undecidable.

Hint: You can use a many-one reduction.

(b) Given natural numbers $x \geq 3$ and $y \geq 3$, we define $f(x, y)$ to be the maximum number of steps that any Turing machine with x states and y different tape characters can take on input ε before halting. (In other words, there is some Turing machine with x states and y different tape characters which, on input ε, takes $f(x, y)$ steps and then halts. Furthermore, every Turing machine with x states and y different tape characters that takes more than $f(x, y)$ steps on input ε will run forever on input ε.) Note that $f(x, y)$ exists because there are only a finite number of different Turing machines that have x states and y different tape characters.

Let $Beaver = \{\langle x, y, f(x, y)\rangle : x \geq 3, y \geq 3\}$. Prove that the language $Beaver$ is undecidable.

Hint: Do not use a many-one reduction; instead show that a subroutine that decides $Beaver$ could be used to compute $f(x, y)$, and then use part (a).